An engineering approach to the source terms treatment in shallow water equations

by

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The long running activity of the research group in the University of Ferrara was focused on the efficient numerical treatment of the shallow water source terms for applications of engineering interest. Both fixed and movable bed problems were considered.

Taking into account the expertise of group members (all engineers) is not surprising if the developed approaches are often based on physically reasoning rather than on a more formal mathematical approach. For example: the well-balanced DFB technique (Divergence Form for Bed Slope Source Term) is based on the idea that the bed slope source term corresponds to the static force due to the non-horizontality of the bottom, and therefore may be computed, in any cell, for a constant value of the free surface elevation [5]; the method for managing the bed discontinuities presented in [2] is based on a suitable reconstruction of the conservative variables at the cell interfaces, coupled with a correction of the numerical flux, based on the local conservation of the total energy (i.e. taking inspiration from the approach usually adopted for the reconstruction of the free surface profile in open channels).

The need of a greater computational efficiency has prompted the use of high-order schemes. For this reason, the topic of the well-balancing was addressed in the context of the central WENO schemes in the works [4, 3] on fixed and movable bed, respectively. In these works suitable procedures are used to achieve the evaluation of the point-values of the flux derivative coupled with the bed slope source term and the spatial integration of the source term itself, analytically manipulated to take advantage from the regularity of the free-surface elevation.

Further difficulties usually arising in the practical applications are due to the necessity of a correct boundary condition modeling. This aspect is challenging in the high-order models also because of the large extension of the computational support (i.e. the stencil) and therefore the suitability of the compact WENO family of Hermitian type is explored in [1]. For the first time this kind of scheme was used to integrate the shallow water equations. The Hermitian WENO scheme, which is interesting for one dimensional problems, becomes too cumbersome in a two dimensional contest, so the very recent activity is finally shifted to the study of third-order 2D RKDG schemes.

It is interesting to note that even if the nature of schemes on which we worked was radically changed during the research (under the pressure of new constrains that gradually are emerged) it was possible to continuously adapt the previously developed techniques to this changing framework.

[1] Caleffi V., A new well-balanced Hermite weighted essentially non-oscillatory scheme for shallow water equations, 2010, International Journal For Numerical Methods In Fluids, accepted, DOI: 10.1002/fld.2410

[2] Caleffi V., Valiani A., Well-balanced bottom discontinuities treatment for high-order shallow water equations WENO scheme, ASCE Journal of Engineering Mechanics, 2009, Vol. 135(7), pp. 684-696.

[3] Caleffi V., Valiani A., Bernini A., High-order balanced CWENO scheme for movable bed shallow water equations, Advances in Water Resources, 2007, Vol. 30(4), pp. 730-741.

[4] Caleffi V., Valiani A., Bernini A., Fourth-order balanced source term treatment in central WENO schemes for shallow water equations, Journal of Computational Physics, 2006, Vol. 218(1), pp. 228-245.

[5] Valiani A., Begnudelli L., Divergence form for bed slope source term in shallow water equations, ASCE Journal of Hydraulic Engineering, 2006, 132(7), 652-665.