Numerical simulation of sediment transport in shallow water equations

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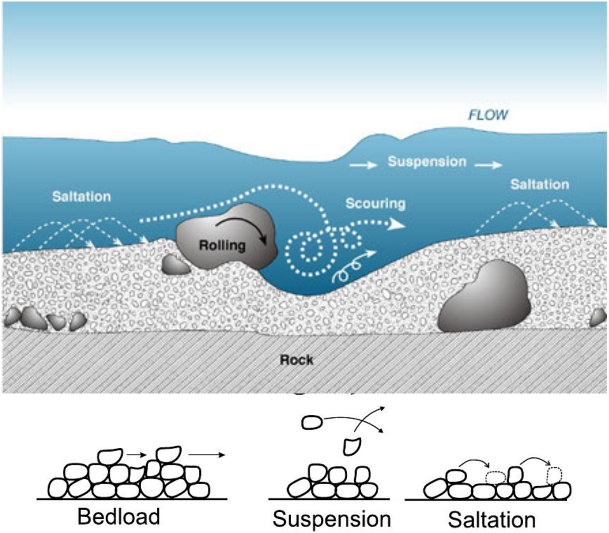
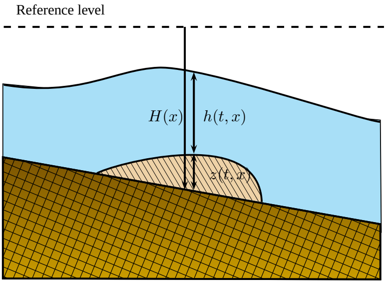
Grupo EDANYA

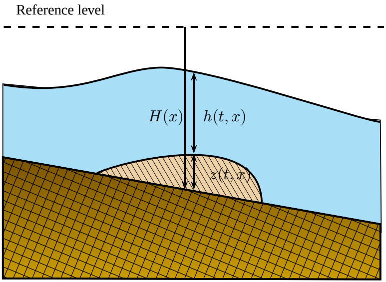
Outline

Outline

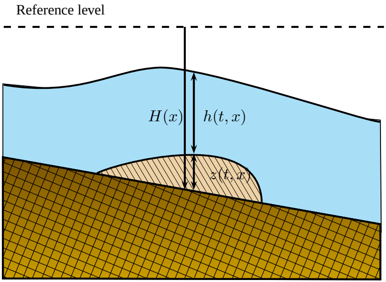

FLOW
Suspension
Saltation
Saltation
Scouring
Rolling
Rock
Bedload
Suspension
Saltation

- Profound impact on the morphology of continental shelves
- Deposit $\Rightarrow$ porous layer of rock $\Rightarrow$ potential sources of hydrocarbon
- Destructive effect (pipelines, cables, fundations,...)


Reference level
$H(x)$
$h(t,x)$
$z(t,x)$



$$\begin{cases} \partial_t h + \partial_x(hu) = 0, \\ \partial_t(hu) + \partial_x(hu^2 + g/2h^2) = gh\partial_x(H - z), \\ \end{cases}$$



$$\begin{cases} \partial_t h + \partial_x(hu) = 0, \\ \partial_t(hu) + \partial_x(hu^2 + g/2h^2) = gh\partial_x(H - z), \\ \partial_t z + \xi\partial_x q_b = 0 \end{cases}$$

Outline

Grass Model

$$q_b(h, hu) = A_g u|u|^{m_g - 1}, \quad 1 \leq m_g \leq 4$$

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- Critical shear stress set to zero

Bottom shear stress

$$\tau_b = \rho_w g h S_{\mathrm{f}},$$

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Darcy-Weisbach

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Manning

$$S_{\mathrm{f}} = \frac{n^2 u|u|}{h^{4/3}}$$

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Darcy-Weisbach

$$\tau_b = \rho_{\mathrm{w}} \frac{f}{8} u|u|$$

Manning

$$\tau_b = g \rho_{\mathrm{w}} \frac{n^2 u|u|}{h^{1/3}}$$

Shield's parameter

$$\tau_b^\star = \frac{\tau_b}{(\rho_s - \rho_{\mathrm{w}})g d_s}$$

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Bedload transport flux

$$q_b = \sqrt{\left(\frac{\rho_\mathrm{s}}{\rho_\mathrm{w}} - 1\right) g d_\mathrm{s}^3}\, \Phi\, \mathrm{sgn}\,(u)$$

Meyer-Peter&Müller model (1948)

$$q_b(h, hu) = \sqrt{\left(\frac{\rho_\mathrm{s}}{\rho_\mathrm{w}} - 1\right) g d_\mathrm{s}^3} \cdot \Phi \cdot \mathrm{sgn}\,(u) \qquad \text{(MPM)}$$

$$\Phi = 8(|\tau_b^\star| - \tau_{cr}^\star)_+^{3/2}$$

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- More complex but gives good results
- Takes into account shear stress ($\tau_{crit}^\star$)
- Valid for slopes lower than 2%

Fernández Luque & Van Beek (1976)

$$q_b(h, hu) = \sqrt{\left(\frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{w}}} - 1\right) g d_{\mathrm{s}}^3} \cdot \Phi \cdot \mathrm{sgn}\,(u)$$

$$\Phi = 5.7(|\tau_b^\star| - \tau_{cr}^\star)_+^{3/2}$$

Ribberink (1998)

$$q_b(h, hu) = \sqrt{\left(\frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{w}}} - 1\right) g d_{\mathrm{s}}^3} \cdot \Phi \cdot \operatorname{sgn}(u)$$

$$\Phi = 11(|\tau_b^\star| - \tau_{cr}^\star)_+^{1.65}$$

Nielsen (1992)

$$q_b(h, hu) = \sqrt{\left(\frac{\rho_{\mathrm{s}}}{\rho_{\mathrm{w}}} - 1\right) g d_{\mathrm{s}}^3} \cdot \Phi \cdot \mathrm{sgn}\,(u)$$

$$\Phi = 12\sqrt{|\tau_b^\star|}(|\tau_b^\star| - \tau_{cr}^\star)_+$$

Outline

Is the model hyperbolic?

$$\begin{cases} \partial_t h + \partial_x(hu) = 0, \\ \partial_t(hu) + \partial_x(hu^2 + g/2h^2) = -gh\partial_x z, \\ \partial_t z + \xi\partial_x q_b = 0 \end{cases}$$

Is the model hyperbolic?

$$\partial_t W + A(W)\partial_x W = 0$$

where

$$W = (h, hu, z)^t, \quad A(W) = \begin{pmatrix} 0 & 1 & 0 \\ gh - u^2 & 2u & gh \\ \dfrac{\partial q_b}{\partial h} & \dfrac{\partial q_b}{\partial q} & 0 \end{pmatrix}$$

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- Write characteristic polynomial:

$$p(\lambda) = \lambda^3 + a_2\lambda^2 + a_1\lambda + a_0$$

$$a_2 = -2u, \quad a_1 = u^2 - gh(1+b), \quad a_0 = ghub, \quad b = \xi\frac{\partial q_b}{\partial(hu)} > 0$$

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- Use Cardano-Vieta relations $Q^3 + R^2 < 0$,

$$Q = \frac{3a_1 - a_2^2}{9}, \quad R = \frac{9a_1a_2 - 27a_0 - 2a_2^3}{54}.$$

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- . . . after some tedious calculations . . .

$$Q^3 + R^2 < 0 \Longleftrightarrow 4h(u^2 - gh)^2 + ghb(14 + d) + 4g^2h^3(b^3 + 3b^2 + 3b) > 0$$

- Same approach could be use for other bedload transport formulae . . .

- Same approach could be use for other bedload transport formulae ...
- ... not an easy task!

Different approach

S. Cordier, M. Le and TML (2011)

Different approach

- Grass:

$$\frac{\partial q_b}{\partial h} = -\frac{q}{h}\frac{\partial q_b}{\partial q}$$

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Manning : $\tau_b = \rho_w g h \frac{n^2 u|u|}{h^{4/3}} = \alpha \frac{u|u|}{h^{1/3}} \quad (\alpha = \text{cst})$

$$\frac{\partial q_b}{\partial h} = -\frac{7}{6}\frac{q}{h}\frac{\partial q_b}{\partial q}$$

Most of the classical formulae for bedload transport satisfy:

$$\frac{\partial q_b}{\partial h} = -k\frac{q}{h}\frac{\partial q_b}{\partial q}.$$

$$A(W) = \begin{pmatrix} 0 & 1 & 0 \\ gh - u^2 & 2u & gh \\ -kub & b & 0 \end{pmatrix}, \quad b = \frac{\partial q_b}{\partial q}$$

$$A(W) = \begin{pmatrix} 0 & 1 & 0 \\ gh - u^2 & 2u & gh \\ -kub & b & 0 \end{pmatrix}, \quad b = \frac{\partial q_b}{\partial q}$$

$$\begin{aligned} p(\lambda) = & \; -\lambda \begin{vmatrix} -\lambda & 1 \\ -u^2 + gh & 2u - \lambda \end{vmatrix} - gh \begin{vmatrix} -\lambda & 1 \\ -kub & b \end{vmatrix} \\ = & \; -\lambda[(u-\lambda)^2 - gh)] + ghb(\lambda - ku) \end{aligned}$$

$$\lambda[(u-\lambda)^2 - gh)] = ghb(\lambda - ku)$$

$$\underbrace{\lambda[(u-\lambda)^2 - gh)]}_{f(\lambda)} = ghb(\lambda - ku)$$

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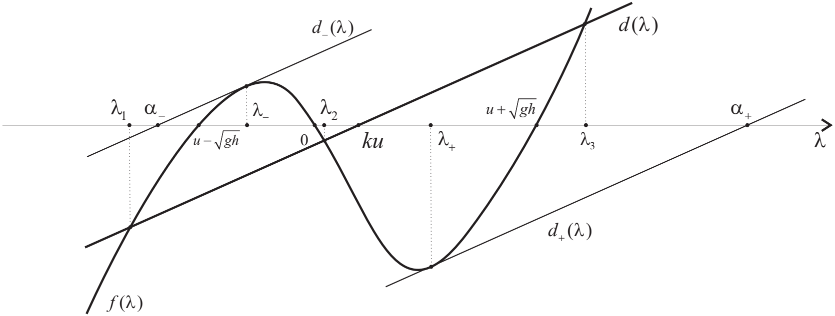
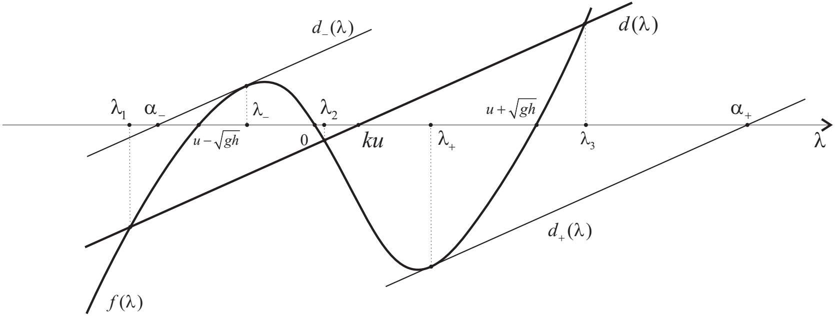
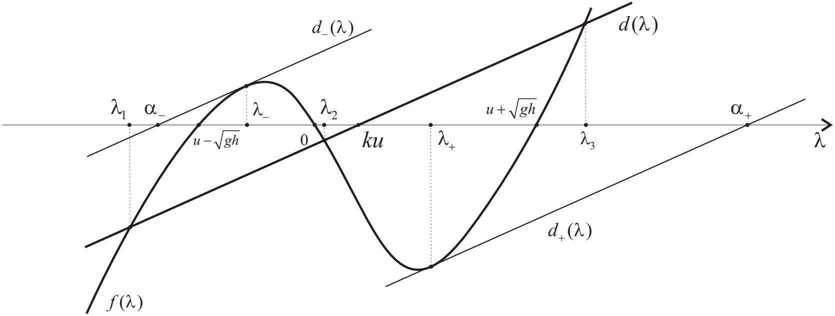
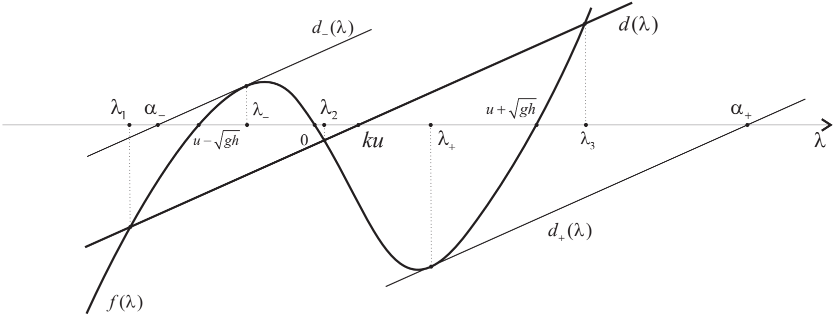




$d_-(\lambda)$
$d(\lambda)$
λ_1
α_-
λ_-
λ_2
$u+\sqrt{gh}$
α_+
$u-\sqrt{gh}$
0
ku
λ_+
λ_3
λ
$d_+(\lambda)$
$f(\lambda)$



- $k = 1 \Rightarrow 3$ eigenvalues always



- $k = 1 \Rightarrow 3$ eigenvalues always
- $k \neq 1 \Rightarrow \alpha_- < ku < \alpha_+$

Theorem

Suppose

$$\frac{\partial q_b}{\partial h} = -k\frac{q}{h}\frac{\partial q_b}{\partial q}.$$

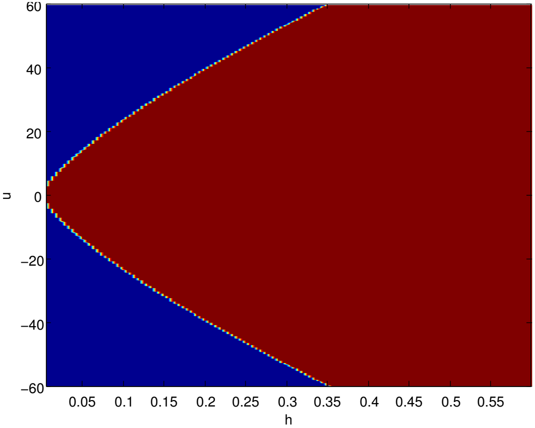
The system is hyperbolic if and only if

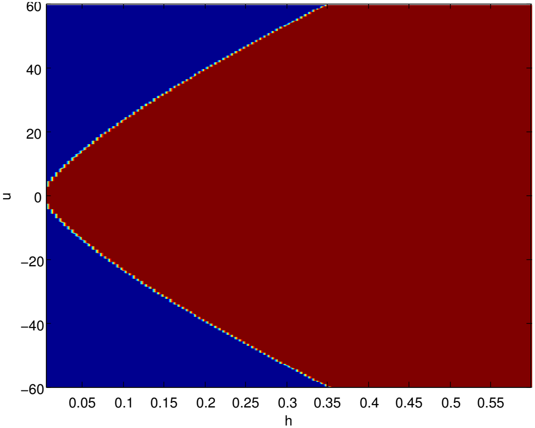
$$\alpha_- < ku < \alpha_+,$$

where

$$\alpha_\pm \stackrel{def}{=} \lambda_\pm - \frac{f(\lambda_\pm)}{ghb}$$

$$\lambda_\pm \stackrel{def}{=} \frac{2u \pm \sqrt{u^2 + 3(gh + ghb)}}{3}.$$

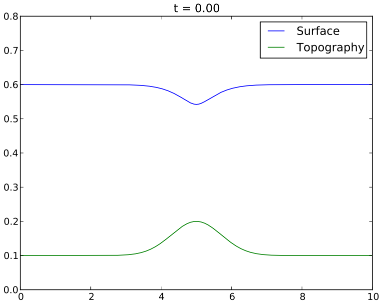

60
40
20
0
−20
−40
−60
u
0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5 0.55
h



In the case $k = 7/6$ a sufficient condition is

$$|u| < 6\sqrt{gh}.$$

Be careful with splitting!



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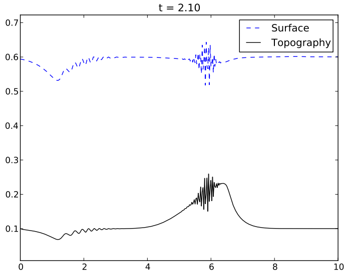

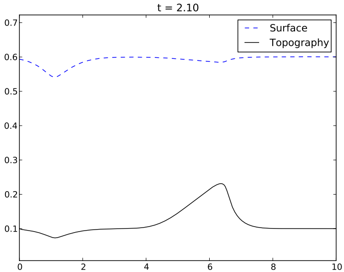

$\text{CFL} = 0.95$

$\text{CFL} = 0.5$

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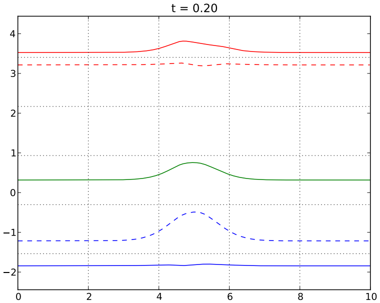

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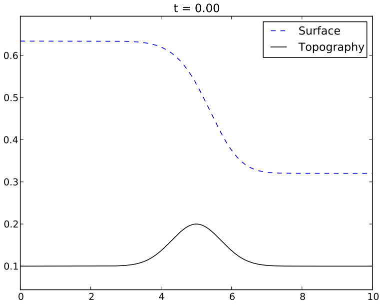

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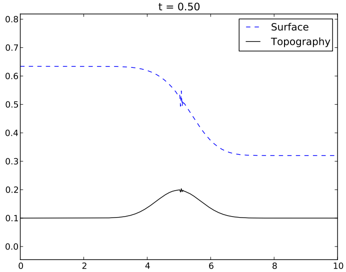

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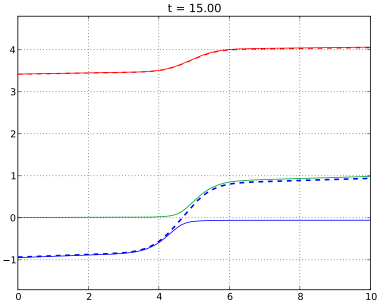

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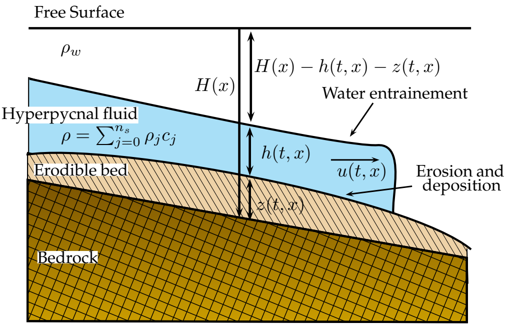
Turbidity current / Hyperpicnal plume



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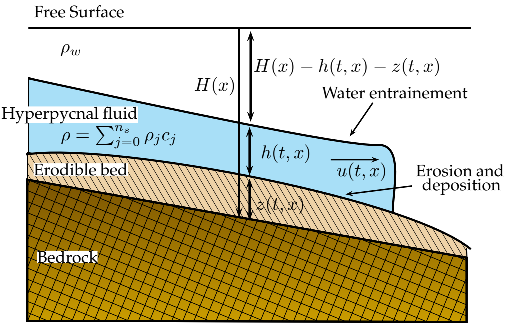

$$
\begin{cases}
\partial_t h + \partial_x(hu) = \phi_\eta + \phi_b, \\
\partial_t(hu) + \partial_x\left(hu^2 + g\,(R_0 + R_c)\,\frac{h^2}{2}\right) = \\
\qquad\qquad g\,(R_0 + R_c)\,h\partial_x(H - z) + u\phi_\eta + \frac{u}{2}\phi_b + \tau, \\
\partial_t(hc_j) + \partial_x(hu\,c_j) = \phi_b^j, \ \text{for } j = 1, \ldots, n_s \\
\partial_t z + \xi\partial_x q_b = -\xi\phi_b.
\end{cases}
$$

Hyperpycnal model

$$
\left\{
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\end{array}
\right. \tag{H}
$$

$$
R_j = \frac{\rho_j - \rho_0}{\rho_0}, \ \text{for } j = 1, \ldots, n_s; \quad R_0 = \frac{\rho_0 - \rho_w}{\rho_0}; \quad \text{and } R_c = \sum_{j=1}^{n_s} R_j c_j.
$$

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Water entrainement

$$
\phi_\eta = E_w u,
$$

$$
E_w = \frac{0.00153}{0.0204 + \mathcal{Ri}}, \qquad \mathcal{Ri} = \frac{R_c g h}{u^2}.
$$

Hyperpycnal model

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Erosion/deposition

$$\phi_b^j = F_e^j - F_d^j, \quad \phi_b = \sum_{j=1}^{n_s} \phi_b^j$$

Polydisperse sedimentation model

N especies of sedidemnt
ϕ_i = volumetric concentration $i = 1, \ldots, N$

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Polydisperse sedimentation model

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$$\rho(\Phi) = \sum_{i=0}^{N} \rho_i \phi_i$$

Polydisperse sedimentation model

$$\partial_t \phi_i + \nabla(\phi_i v_i) = 0, \quad i = 0, 1, \ldots, N,$$

$$\rho_i(\partial_t(\phi_i v_i) + \nabla \cdot (\phi_i v_i \otimes v_i))$$
$$= -\rho_i \phi_i g \vec{k} - \phi_i \nabla p + \alpha_i(\phi)\Delta v_i + m_i^s + \nabla \cdot T_i^E - \nabla \left(\frac{\phi_i}{\phi} \sigma_e(\phi) \right), \; i = 1, \ldots, N,$$

$$\rho_0(\partial_t(\phi_0 v_0) + \nabla \cdot (\phi_0 v_0 \otimes v_0)) = -\rho_0 \phi_0 g \vec{k} \sum_{i=1}^{N} \alpha_i(\phi)\Delta v_i + \nabla \cdot T_0^E,$$

$v_i = (u_i, w_i) \in \mathbb{R}^2$ is the phase velocity
T_i^E viscous stress tensor
m_i^s interaction forces between solid particles
σ_i^E effective solid stress
α_i resistence coefficient for the transfer of momentum

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$$\rho_i(\partial_t(\phi_i v_i) + \nabla \cdot (\phi_i v_i \otimes v_i))$$
$$= -\rho_i \phi_i g \vec{k} - \phi_i \nabla p + \alpha_i(\phi) \Delta v_i + m_i^s + \nabla \cdot T_i^E - \nabla \left(\frac{\phi_i}{\phi} \sigma_e(\phi) \right), \ i = 1, \ldots, N,$$

$$\rho_0(\partial_t(\phi_0 v_0) + \nabla \cdot (\phi_0 v_0 \otimes v_0)) = -\rho_0 \phi_0 g \vec{k} \sum_{i=1}^{N} \alpha_i(\phi) \Delta v_i + \nabla \cdot T_0^E,$$

$$q = \sum_{i=0}^{N} \phi_i v_i$$

$$\Delta v_i = v_i - v_0, \quad i = 1, \ldots, N.$$

Polydisperse sedimentation model

Following Berres et al. (2003) and assuming
$\partial_t(\phi_i w_i) + \partial_x(\phi_i w_i u_i) + \partial_z(\phi_i w_i^2)$ and T_i^E small

Polydisperse sedimentation model

Following Berres et al. (2003) and assuming $\partial_t(\phi_i w_i) + \partial_x(\phi_i w_i u_i) + \partial_z(\phi_i w_i^2)$ and T_i^E small

$$\Delta w_i = \mu \delta_i V(\phi) \left((\overline{\rho}_i - \sum_{j=1}^{N} \overline{\rho}_j \phi_j) + \frac{\sigma_e(\phi)}{g\phi_i} \partial_z \left(\frac{\phi_i}{\phi} \right) + \frac{1-\phi}{g\phi} \partial_z \sigma_e(\phi) \right)$$

$$V(\phi) = (1-\phi)^{n-2},\ n > 2.$$

$\overline{\rho_i} = \rho_i - \rho_0$ for $i = 1, \ldots, N$, $\mu = -g\dfrac{d_1^2}{18\mu_f}$ $\delta_i = \dfrac{d_i^2}{d_1^2}$

MLB model

$$\begin{cases} \partial_t \phi_i + \partial_x(\phi_i u_i) + \partial_z(\phi_i w + f_i(\phi)) = \partial_z(a_i(\phi, \partial_z \phi)), \quad i = 1, \ldots, N \\ \nabla \cdot q = 0 \\ \nabla p = -\nabla \sigma_e(\phi) - \rho(\phi) g \vec{k} + \frac{1}{1-\phi} \nabla \cdot T_0^E \end{cases}$$

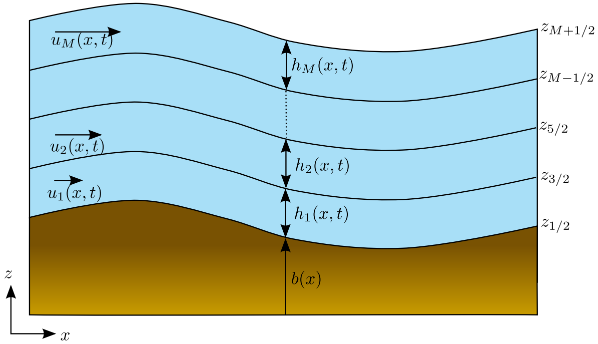
$$f_i(\phi) = \mu V(\phi)\phi_i \left(\delta_i(\overline{\rho}_i - \sum_{j=1}^{N} \overline{\rho}_j \phi_j) - \sum_{k=1}^{N} \delta_k \phi_k \left(\overline{\rho}_k - \sum_{j=1}^{N} \overline{\rho}_j \phi_j \right) \right),$$

MLB model for a one-dimensional closed vessel

$$\partial_t \phi_i + \partial_z (f_i(\phi)) = \partial_z (a_i(\phi, \partial_z \phi)), \quad i = 1, \dots, N$$

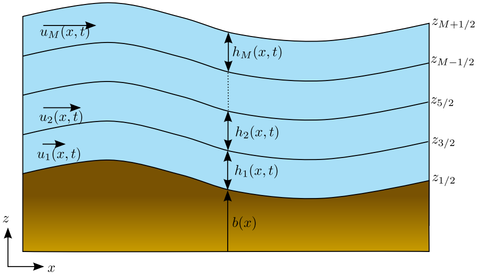
$$f_i(\phi) = \mu V(\phi)\phi_i \left(\delta_i (\overline{\rho}_i - \sum_{j=1}^{N} \overline{\rho}_j \phi_j) - \sum_{k=1}^{N} \delta_k \phi_k \left(\overline{\rho}_k - \sum_{j=1}^{N} \overline{\rho}_j \phi_j \right) \right),$$

A multilayer approach



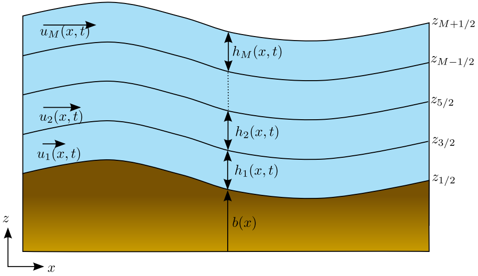
Ongoing work with E.D. Fernández Nieto, E.H. Koné and R. Bürger

A multilayer approach



$$\left\{\begin{array}{l}\partial_t h_\alpha + \partial_x (h_\alpha u_\alpha) = G_{\alpha+\frac{1}{2}} - G_{\alpha-\frac{1}{2}} - F_{\alpha+\frac{1}{2}}(\Phi) + F_{\alpha-\frac{1}{2}}(\Phi),\\ \partial_t (h_\alpha\, \rho_\alpha(\Phi)\, u_\alpha) \;+\; \partial_x (h_\alpha\, \rho_\alpha(\Phi)\, u_\alpha^2) \;+\; \partial_x (h_\alpha\, p_{T,\alpha}) \\ = \displaystyle\sum_{i=0}^{N} \rho_i \left(u_{\alpha+\frac{1}{2}}\, H_{i,\alpha+\frac{1}{2}}(\Phi) - u_{\alpha-\frac{1}{2}}\, H_{i,\alpha-\frac{1}{2}}(\Phi)\right) + \; p_{T,\alpha+\frac{1}{2}} \partial_x z_{\alpha+\frac{1}{2}} \;-\; p_{T,\alpha-\frac{1}{2}} \partial_x z_{\alpha-\frac{1}{2}},\\ \partial_t (h_\alpha \phi_{i,\alpha}) \;+\; \partial_x (h_\alpha \phi_{i,\alpha} u_\alpha) = H_{i,\alpha+\frac{1}{2}}(\Phi) \;-\; H_{i,\alpha-\frac{1}{2}}(\Phi), \quad i = 1, \ldots, N.\end{array}\right.$$

A multilayer approach



$$
\begin{cases}
\partial_t h_\alpha + \partial_x(h_\alpha u_\alpha) = G_{\alpha+\frac{1}{2}} - G_{\alpha-\frac{1}{2}} - F_{\alpha+\frac{1}{2}}(\Phi) + F_{\alpha-\frac{1}{2}}(\Phi), \\
\partial_t(h_\alpha\, \rho_\alpha(\Phi)\, u_\alpha) \;+\; \partial_x(h_\alpha\, \rho_\alpha(\Phi)\, u_\alpha^2) \;+\; \partial_x(h_\alpha\, p_{T,\alpha}) \\
= \displaystyle\sum_{i=0}^{N} \rho_i \left(u_{\alpha+\frac{1}{2}}\, H_{i,\alpha+\frac{1}{2}}(\Phi) - u_{\alpha-\frac{1}{2}}\, H_{i,\alpha-\frac{1}{2}}(\Phi) \right) + \; p_{T,\alpha+\frac{1}{2}} \partial_x z_{\alpha+\frac{1}{2}} \;-\; p_{T,\alpha-\frac{1}{2}} \partial_x z_{\alpha-\frac{1}{2}}, \\
\partial_t(h_\alpha \phi_{i,\alpha}) \;+\; \partial_x(h_\alpha \phi_{i,\alpha} u_\alpha) = H_{i,\alpha+\frac{1}{2}}(\Phi) \;-\; H_{i,\alpha-\frac{1}{2}}(\Phi), \quad i = 1, \ldots, N.
\end{cases}
$$

A multilayer approach

$$\left\{\begin{array}{l}\partial_t h_\alpha + \partial_x(h_\alpha u_\alpha) = G_{\alpha+\frac12} - G_{\alpha-\frac12} - F_{\alpha+\frac12}(\Phi) + F_{\alpha-\frac12}(\Phi),\\ \partial_t(h_\alpha\,\rho_\alpha(\Phi)\,u_\alpha) \;+\; \partial_x(h_\alpha\,\rho_\alpha(\Phi)\,u_\alpha^2) \;+\; \partial_x(h_\alpha\,p_{T,\alpha})\\ = \displaystyle\sum_{i=0}^{N}\rho_i\Big(u_{\alpha+\frac12}\,H_{i,\alpha+\frac12}(\Phi) - u_{\alpha-\frac12}\,H_{i,\alpha-\frac12}(\Phi)\Big) + \; p_{T,\alpha+\frac12}\partial_x z_{\alpha+\frac12} \;-\; p_{T,\alpha-\frac12}\partial_x z_{\alpha-\frac12},\\ \partial_t(h_\alpha\phi_{i,\alpha}) \;+\; \partial_x(h_\alpha\phi_{i,\alpha}u_\alpha) = H_{i,\alpha+\frac12}(\Phi) \;-\; H_{i,\alpha-\frac12}(\Phi), \quad i=1,\dots,N.\end{array}\right.$$

$$G_{\alpha+\frac12} \;:=\; \partial_t z_{\alpha+\frac12} \;+\; u_{\alpha+\frac12}\partial_x z_{\alpha+\frac12} \;-\; w_{\alpha+\frac12} \;\text{ for }\; \alpha = 0,1,...,M$$

$$H_{i,\alpha+\frac12} \;:=\; \phi_{i,\alpha+\frac12}G_{\alpha+\frac12} \;-\; f_{i,\alpha+\frac12}(\Phi) \;\text{ for }\; \alpha = 0,1,...,M,\; i = 0,1,...,N$$

$$p_{T,\alpha+\frac12} \;=\; p_S \;+\; \sum_{\beta=\alpha+1}^{M} h_\beta\,\rho_\beta(\Phi)\,g$$

$$p_{T,\alpha}(t,x) = p_{T,\alpha+\frac12} \;+\; \frac{1}{2}h_\alpha\,\rho_\alpha(\Phi)\,g$$

$$F_{\alpha+\frac12}(\Phi) := \sum_{i=1}^{N} f_{i,\alpha+\frac12}(\Phi)$$

$$h_\alpha = l_\alpha\,h \;\text{ with }\; \sum l_\alpha = 1$$

Thank you for your attention

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