# INTAS PROPOSAL FOR
# Open Call 2003 - Research Project

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**1.1 TITLE:**
**Simplicial algebra, homology theories, K-theory and homotopy theory**

**1.1.1** **Keyword 1 : Algebra**
**Keyword 2 : Topology & Manifolds**
**Keyword 3 : Geometry, Algebraic Geometry**

**1.1.2** **Free word 1 : Homological and homotopical algebra**
**Free word 2 : Homology properties of manifolds**
**Free word 3 : Cohomology of groups and K-theory**

**1.1.3** **Intended Start Date: January 2004**

**1.1.4** **Duration: 36 Months**

**1.2 CONSORTIUM**

Université Paris 13, - France
A.Razmadze Mathematical Institute - Georgia
Université catholique de Louvain - Belgium
University of Glasgow - United Kingdom
University Montpellier 2 - France
Sobolev Institute of Mathematics - Russia
St-Petersburg State University - Russia

**1.3 SUMMARY**

The project is centered around simplicial algebra and topology, (co)homology theories, algebraic K-theory and cobordism. It is intended to make progress on the Karoubi conjecture, to develop further the interrelation between algebraic K-theory, bivariant K-theory and equivariant homology of groups, to construct and develop the non- abelian cohomology of crossed structures; another goal is to construct and study the n-fold Cech cohomology of open covers and the n-fold Cech derived functors of group valued functors.

A second aspect of the program is to produce new information on the multiplicative structure of the simplectic cobordism ring, to give presentations of Morava K-theory and Brown-Peterson cohomology of p-groups in terms of transferred Chern classes, to get new calculations for elliptic cohomology for toric manifolds, and to obtain the analog of the Birman Ko Lee presentation for singular braid monoid.

The third aspect is to find conditions of triviality for the second L_p-cohomology of discrete groups with applications to noncompact manifolds, to obtain analogues to the Davis-Okun theories for right-angled Coxeter groups, to determine the homotopy type of embedding spaces of manifolds. Related to K-theory and Hochschild cohomology
one wants computations of the Gerstenhaber algebra on the homology of the free loop space.

One wants to make a significant contribution towards the proof of the Milnor-Friedlander conjecture, to prove the rigidity of the Henselian case for all

cohomologies represented by a T-spectrum, to calculate cohomology of Steinberg groups modeled on Chevalley groups.

The last aspect of the program concerns explicit computations of primary cyclic and Hochschild homologies for commutative algebras, applications to representation theory of algebras, developing the foundation of the theory of exact couples in Raikov-semiabelian categories, and the study of the global properties of certain functor categories : the so called "artinian conjecture" as well as their Gabriel-Krull filtration.

# 2 TEAM INFORMATION

## 2.1 Team : Paris 13 Team

### 2.1.1 Team Description

The team of Paris 13 has an expertise in Hochschild homology, K-theory, cohomology of functors, and the structure of polynomial functors. In particular M. Vigue and has worked on the cohomology of the free loop spaces and the growth of their Betti numbers. She made with
D. Sullivan conjectures on this subject that are
not yet up to now completly proved. Recently there has been some advances in this field: Chas and Sullivan defined a Gerstenhaber algebra structure on the cohomology of the free loop space of a compact manifold. M. Vigue has determined with J. C. Thomas an algebraic model to compute this structure. M. Vigue has also worked on the properties of commutative algebras in term of Hochschild homology. In particular she has caracterised complete intersections.

M. Karoubi is a great expert both in topological and algebraic in K-theory and in fact one of those at the origin of algebraic K-theory. He is presently working on hermitian K-theory, the Baum-Connes conjecture for real C^* algebras. He is looking for integral minimal "braided" model for the homotopy type of spaces.

G. Powell and L. Schwartz have both worked intensively in the theory of unstable modules over the Steenrod algebra, the structure of polynomial functors and Mac Lane cohomology. G. Powell is also working on the motivic Steenrod algebra.

L. Schwartz (with students) is studying (mainly using the Eilenberg-Moore spectral sequence) the question of the realization of unstable modules as cohomology of spaces and functors over quadratic spaces.

The field of M. Livernet cocerns operads and various applications of the theory.

Thus, the team of Paris 13, taking also into account, other people in the mathematic departement offers a wide expertise concerning homology theories in a broad sense. It is not only composed of people of Paris 13 because it does not make sense to create 3 different teams, all the people in the team are either very close geographically and/or scientifically and in any case meet very often.

### 2.1.2 List of publications


1 Finitness conditions for Hochschil homology algebra and the free loop space cohomology algebra, L. Piriou & L. Schwartz, K-theory 21 (2002).
2 Hochschild homology criteria for trivial algebras, M. Vigue, Trans. Am. Math. Soc. 354 (2002).
3 On a plus-construction for algebras over an operad, M. Livernet, K-theory 18 (1999).
4 The structure of the tensor product of F[-] with a finite functor between vector spaces, G. Powell, Ann. Inst. Fourier, 50 (2000).
5 Algebraic braided model of the affine line and difference calculus on a topological space, M. Karoubi, Comptes Rendus Académie Sciences Paris, t. 335, Sér. I (2002), p. 121-126.


### 2.1.3 Team Leader and address

| | |
|---|---|
| Title | Prof. |
| Position | Head of laboratory |
| Sex | Male |
| Date Of Birth | 05/07/1953 |
| First Name | Lionel |
| Patronic Name | |
| Family Name | Schwartz |
| Organisation Type | Public |
| Organisation Registration Nr. | |
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| Website | http://zeus.math.univ-paris13.fr/ |

### 2.1.4 List of Senior Scientists in the team

1) First Name Max
   Patronic Name
   Family Name Karoubi
   Year Of Birth 1938
   Insitute Université Paris 7
2) First Name Muriel
   Patronic Name
   Family Name Livernet
   Year Of Birth 1972
   Insitute Université Paris 13
3) First Name Geoffrey

| | |
|---|---|
| Patronic Name | |
| Family Name | Powell |
| Year Of Birth | 1969 |
| Insitute | Université Paris 13 |
| 4) First Name | Jean Claude |
| Patronic Name | |
| Family Name | Thomas |
| Year Of Birth | 1948 |
| Insitute | Université d'Angers |
| 5) First Name | Micheline |
| Patronic Name | Poirrier |
| Family Name | Vigue |
| Year Of Birth | 1948 |
| Insitute | Université Paris 13 |

### 2.1.5 Statistics

Number of Team Members involved in this project: 6
Number of Team Members under 35: 2
Number of Team Members who have individually received grants in INTAS projects: 0

## 2.2 Team : Tbilisi Team

### 2.2.1 Team Description

A.Razmadze Mathematical Institute,
Georgian Academy of Sciences.
Malkhaz Bakuradze, Hvedri Inassaridze (local coordinator, Academician of the Georgian Academy of Sciences), Nick Inassaridze (32 years), Tamaz Kandelaki, Emzar Khmaladze (32 years).
Tbilisi team is experienced in homotopical and homological algebra, non-abelian (co)homology of groups and Lie algebras, K-theory of normed algebras, cyclic homology, generalized co(homology) theories, characteristic classes, formal group laws, transfers.

### 2.2.2 List of publications

1 K-theory of stable generalized operator algebras, H.Inassaridze & T.Kandelaki, K-Theory 27 (2002), 103-110.
2 Higher nonabelian cohomology of groups, H.Inassaridze, Glasgow Math. J. 44 (2002), 497-520.
3 N-Fold Cech derived functors and Generalised Hopf type formulas, G.Donadze, N.Inassaridze & T.Porter, Advances Math., 2003 (to appear) (K-theory Preprint Archives: http://www.math.uiuc.edu/K-theory/0624/).
4 Algebraic K-theory of normed algebras, H. Inassaridze, K-Theory 21, No 1 (2000), 25-56.
5 The transfer and symplectic cobordism, M.Bakuradze, Trans. Amer. Math. Soc. 349, no. 11 (1997), 4385--4399.

### 2.2.3 Team Leader and address

| | |
|---|---|
| Title | Prof. |
| Position | Head of department |

| | |
|---|---|
| Sex | Male |
| | |
| Date Of Birth | 06/12/1932 |
| First Name | Hvedri |
| Patronic Name | |
| Family Name | Inassaridze |
| Organisation Type | Public |
| Organisation Registration Nr. | |
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| Organisation / University / Institute | A.Razmadze Mathematical Institute |
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| Laboratory | |
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### 2.2.4 List of Senior Scientists in the team

1) First Name Malkhaz
   Patronic Name
   Family Name Bakuradze
   Year Of Birth 1961
   Insitute A.Razmadze Mathematical Institute
2) First Name Nick
   Patronic Name
   Family Name Inassaridze
   Year Of Birth 1971
   Insitute A.Razmadze Mathematical Institute
3) First Name Tamaz
   Patronic Name
   Family Name Kandelaki
   Year Of Birth 1951

| | |
|---|---|
| Insitute | A.Razmadze Mathematical Institute |

### 2.2.5 Statistics

Number of Team Members involved in this project: 5
Number of Team Members under 35: 2
Number of Team Members who have individually received grants in INTAS projects: 4

## 2.3 Team : LLN Team

### 2.3.1 Team Description

The team of Louvain-La-Neuve has an expertise in Hochschild homology, rational homotopy and geometric applications of homotopy theory. In particular Y. Félix has worked on the cohomology of the free loop spaces and Pascal Lambrechts on the growth of their Betti numbers. Recently there has been some advances in this field: Chas and Sullivan defined a Gerstenhaber algebra structure on the cohomology of the free loop space of a compact manifold. Y. Félix has determined with J. C. Thomas and M. Vigué an algebraic model to compute this structure. Y. Félix has also obtained with J.C. THomas and L. Menichi a duality theorem for topological Hochschild homology.

### 2.3.2 List of publications

1 Rational Homotopy Theory, Y. Félix, S. Halperin and J.C. Thomas, Graduate Texts in Mathematics, 205, Springer-Verlag, 2000.
2 Structure of the loop homology algebra of a closed manifold,
Y. Félix, J.-C. Thomas and M. Vigué,
submitted for publication.
3 Monoid of self-equivalences and free loop spaces, Y. Félix and J.-C. Thomas, Proc. Amer. Math. Soc. (to appear).
4 Embeddings up to homotopy type into Euclidean space, P. Lambrechts, D. Stanley & L. Vandembroucq, Trans. Amer. Math. Soc 354 (2002), 3973-4013.
5 On the rational homotopy type of configurations spaces on two points, P. Lambrechts et D. Stanley, Prépublication 2001.

### 2.3.3 Team Leader and address

| | |
|---|---|
| Title | Prof. |
| Position | Academician |
| Sex | Male |
| Date Of Birth | 13/04/1951 |
| First Name | Yves |
| Patronic Name | |
| Family Name | Félix |
| Organisation Type | Public |
| Organisation Registration Nr. | |
| Academy / Branch | |
| Organisation / University / Institute | Université catholique de Louvain |
| Department | Mathématiques |
| Laboratory | |

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### 2.3.4 List of Senior Scientists in the team

| | |
|---|---|
| 1) First Name | Pascal |
| Patronic Name | |
| Family Name | Lambrechts |
| Year Of Birth | 1964 |
| Insitute | LLN |

### 2.3.5 Statistics

Number of Team Members involved in this project: 2
Number of Team Members under 35: 0
Number of Team Members who have individually received grants in INTAS projects: 0

## 2.4 Team : Glasgow Team

### 2.4.1 Team Description

The Glasgow team with A. Baker, N. Ray, F. Clarke, A. Lazarev, B. Richter, taking also into account, other people in related mathematic departements offers a wide expertise concerning generalized homology theories: cobordism, elliptic cohomology; fine structure on ring spectra and homological technics used in this context. It contains also experts in formal group theory and related arthmetic themes. It is not only composed of people of Glasgow because it does not make sense to create 3 different teams, all the people in the team are either very close geographically and/or scientifically and in any case meet very often.

### 2.4.2 List of publications

1 Brave new Hopf algebras and extension of MU-algebras, A. Baker & A. Jeanneret, HHA 4 (2002).
2 Topological Hochschild cohomology and generalized Morita equivalence, A. Baker & A. Lazarev, Glasgow University Mth. Dept Preprint
02/50.
3 A_infinity structures on some spectra related to Morava K-theory, A. Baker, Quart. J. Math. Oxf. (2), 42 (1991), 403-419.

4 Bockstein operations in Morava K-theory, Forum Math., A. Baker & U. Wuergler, 3 (1991), 543-60.
5 Hecke algebras acting on elliptic cohomology, A. Baker, Contemp. Math. 220 (1998), 17-26.

### 2.4.3 Team Leader and address

| | |
|---|---|
| Title | Dr. |
| Position | Academician |
| Sex | Male |
| Date Of Birth | 02/12/1953 |
| First Name | Andrew |
| Patronic Name | |
| Family Name | Baker |
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| Organisation Registration Nr. | |
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| Website | http://www.maths.gla.ac.uk/~ajb |

### 2.4.4 List of Senior Scientists in the team

1) First Name Francis
   Patronic Name
   Family Name Clarke
   Year Of Birth 1947
   Insitute University of South Wales
2) First Name Martin
   Patronic Name
   Family Name Crossley

| | |
|---|---|
| Year Of Birth | 1969 |
| Insitute | University of South Wales |
| 3) First Name | Andrey |
| Patronic Name | |
| Family Name | Lazarev |
| Year Of Birth | 1968 |
| Insitute | University of Bristol |
| 4) First Name | Nigel |
| Patronic Name | |
| Family Name | Ray |
| Year Of Birth | 1945 |
| Insitute | University of Manchester |
| 5) First Name | Birgit |
| Patronic Name | |
| Family Name | Richter |
| Year Of Birth | 1971 |
| Insitute | Bonn University |

### 2.4.5 Statistics

Number of Team Members involved in this project: 6
Number of Team Members under 35: 3
Number of Team Members who have individually received grants in INTAS projects: 0

## 2.5 Team : Montpellier Team

### 2.5.1 Team Description

The team of Montpellier has an expertise in cobordism and braids theory, cohomologies of categories and algebraic structures, operads, rewriting systems. In particular, J-M. Oudom is one of young french scientist with a good expertise in operads theory and relationship with E-infinity structures, B. Bendiffalah and D. Guin have studied in details the cohomology of triangular algebras in relations with the cohomology of categories of morphisms. D. Guin, and his students Y. Guiraud and Ph. Malbos, use homological methods to study the problem of the rewriting in the general framework of the cohomology of categories.

### 2.5.2 List of publications

1 Characteristic classes and transfer relations in cobordism, M. Bakuradze, M.Jibladze & V.Vershinin, Proc. Amer. Math. Soc., V. 131, No 6 (2003), 1935-1942.

2 Cohomologie des algebres de Lie croisees et K-theorie de Milnor additive, D.Guin, Annales de l'institut Fourier, 45(1995), 93-118.

3 Cohomologie et homologie non abeliennes des groupes, D.Guin, J. Pure Appl. Algebra 50(1988), 109-137.

4 Théorème de Leray dans la catégorie des algébres sur une opérade}, J-M. Oudom, C.R.A.S. Paris Série I, Math. 329 (1999), No 2, 101-106.

5 Homologie du groupe lineaire et K-theorie de Milnor des anneaux, D.Guin, J. of Algebra, 123(1989), 27-59.

### 2.5.3 Team Leader and address

| | |
|---|---|
| Title | Prof. |
| Position | Academician |
| Sex | Male |
| Date Of Birth | 20/04/1946 |
| First Name | Daniel |
| Patronic Name | |
| Family Name | Guin |
| Organisation Type | Public |
| Organisation Registration Nr. | |
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| Organisation / University / Institute | University Montpellier 2 |
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### 2.5.4 List of Senior Scientists in the team

1) First Name Belkacem
   Patronic Name
   Family Name Bendiffalah
   Year Of Birth 1965
   Insitute Université de Montpellier
2) First Name Jean-michel
   Patronic Name

Family Name Oudom
Year Of Birth 1968
Insitute Université de Montpellier
3) First Name Vladimir
Patronic Name
Family Name Vershinin
Year Of Birth 1952
Insitute Université de Montpellier

### 2.5.5 Statistics

Number of Team Members involved in this project: 4
Number of Team Members under 35: 0
Number of Team Members who have individually received grants in INTAS projects: 0

### 2.6 Team : Novosibirsk Team

### 2.6.1 Team Description

Sobolev Institute of Mathematics,
Russian Academy of Sciences (Siberian Branch).
N. Glotko (24 years old), Ya.Kopylov (local coordinator, 27 years old)
The Novosibirsk Team has significant results in the L_p-theory of differential forms on Riemannian manifolds, homological algebra in the categories of Hilbert and Banach spaces and Raikov-semiabelian categories.

### 2.6.2 List of publications

1 1. Some properties of the operator of exterior derivation on surfaces of revolution and L_p-cohomology, Ya.A.Kopylov, Complex Geometry of Groups, Contemp. Math. 240 (1999), 247-257.
2
2. On the K\"unneth formula for the reduced cohomology of the tensor product of complexes of Hilbert spaces, Ya.A.Kopylov, Siberian Math. J. 40 (1999), No.5, 901--906.
3 3. On the Ker-Coker-sequence in a semiabelian category, Ya.A.Kopylov and V.I.Kuz'minov, Siberian Math. J., 41 (2000), No.3, 509-517.
4 4. On the cohomology sequence in a semiabelian category, N.V. Glotko and V.I.Kuz'minov, Sibirsk. Mat. Zh. 43 (2002), No.1, 28--35.
5 5. Exactness of the cohomology sequence for a short exact sequence of complexes in a semiabelian category, Ya.A.Kopylov and V.I.Kuz'minov, Siberian Adv. Math. 13 (2003), No.3

### 2.6.3 Team Leader and address

| | |
|---|---|
| Title | Cand. Sc. |
| Position | Senior scientist |
| Sex | Male |
| Date Of Birth | 29/06/1975 |
| First Name | Yaroslav |
| Patronic Name | Anatolievich |
| Family Name | Kopylov |

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| Organisation Type | Public |
| Organisation Registration Nr. | |
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| Organisation / University / Institute | Sobolev Institute of Mathematics |
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| Website | |

### 2.6.4 List of Senior Scientists in the team

1) First Name Nikolay
   Patronic Name Vladimirovich
   Family Name Glotko
   Year Of Birth 1978
   Insitute Novosibirsk State University

### 2.6.5 Statistics

Number of Team Members involved in this project: 2
Number of Team Members under 35: 2
Number of Team Members who have individually received grants in INTAS projects: 1

## 2.7 Team : St.-Petersburg Team

### 2.7.1 Team Description

St.-Petersburg State University (SPbGU) and the St.-Petersburg Branch of Steklov Institute of Mathematics of the Russian Academy of Sciences (POMI). Local coordinator: Nikolai Vavilov. Senior Scientists: Ivan Panin (corresponding member of the Russian Academy of Sciences), Alexei Stepanov, Sergei Yagunov (34 years). Scientists: Mikhail Mitrofanov (22 years), Nikita Semenov (23 years), Victor Petrov (20 years), Ekaterina Sopkina (21 years). St.-Petersburg team is experienced in algebraic K-theory, generalised homology theories, Riemann-Roch type theorems, the structure, homology and representation theory of algebraic groups, related groups (classical-like groups and groups of Lie type, group schemes) and Lie algebras. The four junior members of the group are Ph.D. students, but already have very important contributions in the field, and have published or submitted 2 to 8 papers each.

### 2.7.2 List of publications

1 Rigidity for Orientable Functors, I.Panin, S.Yagunov, J. Pure Appl. Algebra, 2002, vol.172, 49-47.

2 Decomposition of transvections: Theme with variations, A.Stepanov, N.Vavilov, K-Theory, vol.19, no. 2 (2000), 109-153.
3 K_1 of Chevalley groups are nilpotent, R.Hazrat, N.Vavilov, J.Pure Appl. Algebra, vol.179 (2003), 99-116.
4 A purity theorem for the Witt group, M.Ojanguren, I.Panin, Ann. Sci. Ecole Norm. Sup. 4^eme serie, vol.32, N.1 (1999), 71-86.

5 Push-forwards in oriented cohomology theories of algebraic varieties: II, I.Panin, (to appear),
http://www.math.uiuc.edu/K-theory/0619/2003

### 2.7.3 Team Leader and address

| | |
|---|---|
| Title | Prof. |
| Position | Academician |
| Sex | Male |
| | |
| Date Of Birth | 17/09/1952 |
| First Name | Nikolai |
| Patronic Name | Alexandrovich |
| Family Name | Vavilov |
| Organisation Type | Public |
| Organisation Registration Nr. | |
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| Academy / Branch | |
| Organisation / University / Institute | St-Petersburg State University |
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| Laboratory | Algebra and Number Theory |
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| Website | |

### 2.7.4 List of Senior Scientists in the team

1) First Name Ivan
   Patronic Name Alexandrovich
   Family Name Panin
   Year Of Birth 1959
   Insitute POMI
2) First Name Alexei
   Patronic Name Vladimirovich
   Family Name Stepanov
   Year Of Birth 1962
   Insitute SPbGU
3) First Name Sergei
   Patronic Name Alexeevich
   Family Name Yagunov
   Year Of Birth 1969
   Insitute POMI

### 2.7.5 Statistics

Number of Team Members involved in this project: 8
Number of Team Members under 35: 5
Number of Team Members who have individually received grants in INTAS projects: 4

## 3 OBJECTIVES

### 3.1 RESEARCH OBJECTIVES

**(1)** To develop the mod q Quillen algebraic K-theory of normed algebras.
**(2)** To prove Karoubi's Conjecture on the isomorphism of algebraic and topological K-functors for a wider class of normed algebras.
**(3)** To study the algebraic and topological bivariant K-theories and their relationship with Kasparov KK-theory for Z_2-graded complex and real algebras.
**(4)** To continue the investigation of the equivariant (co)homology of groups.
**(5)** To develop and investigate the notion of n-fold Cech derived functors of group valued functros and to apply it to various homology theories and K-theory.
**(6)** To construct and investigate an n-fold Cech (co)homology of open covers of spaces with coefficients in sheaves of abelian groups.
**(7)** To investigate the higher dimensional non-abelian cohomology of Lie algebras.
**(8)** To construct and study a Taylor tower for functors from the pointed category to the category of groups.
**(9)** To investigate primary Hoshcshild and cyclic (co)homology of commutative algebras, and the relation with the Chas-Sullivan loop product on a manifold.
**(10)** To continue the investigation of interaction of transfers and Chern characteristic classes in complex cobordism and Brown-Peterson cohomology.
**(11)** To consider the transferred Chern classes and in terms of formal group law give more natural presentation of the multiplicative structure in complex oriented cohomology rings of p-groups.
**(12)** To study (co)homology theories represented by a T-spectrum.
**(13)** To generalise some cohomological and K-theoretical results to Chevalley groups.
**(14)** To study the L_p-cohomology of noncompact Riemannian manifolds and discrete groups and the homological algebra of Raikov's semiabelian categories.
**(15)** To study the space of embedding of a manifold into another one using Goodwillie calculus.
**(16)** To use homological algebra to obtain information on the homotopy type of configuration spaces

## 3.2 Background and Justification

The current consortium was formed under similar scientific interests of the teams in simplicial algebra, homology theory, K-theory and homotopy theory and which is reflected in the title of the project. Members of two teams, Tbilisi and St.-Petersburg, of the consortium collaborated and participated in jointly workshops and discussed on some topics of K-theory and homological algebra. Two INTAS teams, Paris 13 and Louvain have a long tradition of cooperation, either directly, either through European networks. The Glasgow team has had regular contact with Paris 13, Tbilisi and Vershinin in Montpellier. All teams of the current consortium have close relationship expressed in many aspects of scientific life.

Three of the leading centers of topology, homological algebra and K-theory in the former Soviet Union were particularly in Tbilisi at the Razmadze Mathematical Institute of the Georgian Academy of Sciences, in St.-Petersburg at the Steklov Mathematical Institute and in Novosibirsk at the Sobolev Institute of Mathematics of the Russian Academy of Sciences which have intensive scientific collaborations to each other and to the leading centers in all over the world and in particular in Europe. This project will help to continue previous scientific cooperation between the teams moreover to intensify them. Some examples of previous cooperation are:

- members of Tbilisi team and St.-Petersburg team participated in jointly workshops and conferences at the University of Bielefeld and the University of Wales (Bangor);
- Bakuradze and Vershinin (with Jibladze) wrote 2 joint papers;
- leaders of Paris 13 team, Glasgow team, Montpellier team and Tbilisi team collaborating in establishing and developing of a new electronic journal Homology, Homotopy and Applications.
- H. and N. Inassaridze visited the University of Montpellier II to discuss with Guin on non-abelian (co)homology and gave 2 talks at the seminar "Algebraic geometry" in 2001.
- Vavilov and Panin visited Paris several times on invitations of Karoubi (member of Paris 13 team).

All team leaders and most of participants of the consortium are well known experts in current fields of mathematics and the research project will capitalize on the expertise of all team leaders. In addition to their other mathematical interests, the members of the seven participating teams have common research interests in (co)homology and homotopy theories, in the more specialized theory of non-abelian (co)homology of groups and Lie algebras, in application of theses theories to algebraic K-theory and cyclic homology . There is also a common interest in a least four teams (Tbilisi, Paris 13, Louvain, Montpellier) to Hochschild homology and/or its applications to the cohomology of the free loop space. Similarly common interest exists in Paris 13, Tbilisi, Montpellier and St-Petersburg in functor categories and functor cohomology, also functor cohomology is known (after the work of Pirashvili and Walhausen, Dundas and Mc Carthy, Friedlander and Suslin) to have deep connections with K-theory and cohomology of group schemes. Cobordism and modern stable homotopy theory concerns mainly Glasgow, Montpellier, Tbilisi and St.-Petersburg, but is also of interest for the other teams in particular for Paris 13 where Powell and Schwartz made some work around cobordism. Tbilisi and St.-Petersburg teams are completing each other on closely related topics of algebraic K-theory and homology theories, Tbilisi team much more develops the general framework, whereas St.-Petersburg team looks at concrete examples.

The proposed network has a wide spectrum of expertise, as said above central in these competences are techniques of homological algebra, simplicial topology, K-theory and the theory of algebraic groups. The addition of an expertise concerning modern stable homotopy theory and in particular elliptic cohomology will bring new perspectives in the picture.

One goal of this project is to promote significantly all the above mentioned common interests, this will greatly benefit the research activity of all eight teams. The list of objectives of the current project exhibits the wide range of interests existing in the consortium. The corresponding pool of techniques, methodologies, and insides knowledge possessed by members will lead to stimulating and productive exchanges of ideas and information among teams through scientific visits, workshops, conferences and the internet.

The main aim of the project is the study of homological and homotopical properties of algebraic and topological structures. A theme common to all of the teams in the consortium is simplicial algebra. Simplicial algebra is an important subject which is being developed by an increasing number of teams all over the world. It has fundamental applications in diverse fields of mathematics and has made a significant impact on the development of many areas of current mathematics. Simplicial algebra, its non-abelian and categorical aspects have played a crucial role in the progress of rapidly expanding areas of K-theory, cyclic homology and homotopy theory.

Karoubi [19] who is one of the very best expert in algebraic K-theory, is also a first rank expert in homotopy theory. H.Inassaridze is a first rank expert in homological algebra and K-theory [10, 11]. The Paris 13 team (Powell, Schwartz [24-26]) has made fundamental contributions to the study of functor categories, functor cohomology, relating functor category to unstable modules over the Steenrod algebra in particular. H. Inassaridze and Kandelaki made substantial contributions in K-theory of normed algebras, in particular towards the investigation of Karoubi's conjecture [18] and in Kasparov bivariant K-theory [11, 14, 16, 17]. Vigue [6, 29, 30], Thomas and Felix made very important contributions to Hochschild homology and the cohomology of the free loop space. N.Inassaridze, Porter and Donadze gave substantial contributions to establishing generalised Hopf type formulae for higher integral homology of groups and for algebraic K-functors [5]. Baker and Ray are first rank experts in cobordism and elliptic cohomology, operations in cohomology theory. Guin, H. and N.Inassaridze made important contributions to non-abelian (co)homology of groups and Lie algebras [7, 8, 12, 13, 15]. Bakuradze and Vershinin have derived transfer methods for investigation of the multiplicative structure in symplectic and self-conjugate cobordism [1, 2]. Bakuradze and Priddy have studied the interaction of transfer and characteristic classes in complex oriented cohomology theories [3, 4]. Panin and Yagunov jointly with Ojanguren [20 -22] obtained important results for generalized (co)homology theories, algebraic cobordism theory and other functors with transfer. Panin gave a very broad generalization of Riemann-Roch-Hirzebruch-Grothendieck theorem. Vavilov jointly with Hazrat [9] successfully studied K_1 for Chevalley groups. Stepanov and Vavilov [28] developed a new geometric approach to the structure theory and K-theory of algebraic groups. Vavilov and Petrov calculated cohomology of Steinberg groups [23].

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## 4 SCIENTIFIC / TECHNICAL DESCRIPTION

### 4.1 Research Programme

As said in the document giving the backgrounds of the network the consortium is formed by teams having broad scientific interests, but most teams use simplicial algebra and/or topology and homological algebra either as central means, either as objects of study themselves. Particulary central in the network are certain techniques concerning non-abelian cohomology, cyclic homology, Hochschild homology, Mac Lane homology, Gamma-homology ("à la Robinson-Whitehouse"). These homology have their own internal interest, however they are, to a very large extent, motivated by their present or potential applications in other fields in of algebraic topology or K-theory. As examples are the applications to algebraic K-theory, cohomology of the free loop spaces, cohomology of Lie algebras, structure of ring spectra. The principal aim of the consortium is to promote cooperation between the various nodes so as the nodes can share their expertise.

Precise tasks and scientific objectives are described in details elsewhere, main title are as follows :

1. Algebraic K-theory
2. Non-abelian cohomology
3. Cohomology of manifolds
4. Generalized cohomology, cobordism
5. Cohomology of algebraic groups
6. Category theory, functor and Hochschild cohomology

Item 2 and 6 are strongly, using related techniques. These techniques have potential applications in items 1, 3, 4 and 6. These last items are source of problems for 3 and 6.

There is going to be all along the duration of the network a certain number of congresses in order to enhance discussion and collaboration. Other initiatives will be taken.

### 4.2 Project Structure

#### 4.2.1 Task Title : Algebraic K-theory

Task coordinator : Hvedri Inassaridze, belonging to team: Tbilisi Team

**Objectives :**
Various aspects of algebraic K-theory will be studied, including the interrelation between algebraic, topological and bivariant K-theories, the connection with equivariant homology theory of groups and their mod q versions.

**Methodology :**

Methods of simplicial algebra, category theory, K-theory, homotopy theory and Banach algebras.

**Task Input:**

The task is depending on : Category theory, functor cohomology, Hochschild homology
This is made precise in the subtasks descriptions.

**Result, milestones :**
See the subtasks descriptions.

#### 4.2.1.1 Task Title : K-theory of normed algebras

Task coordinator : Hvedri Inssaridze, belonging to team: Tbilisi Team

**Objectives :**
(1) To prove Karoubi's Conjecture on the isomorphism of algebraic and topological K-functors for a wider class of normed algebras.
(2) To develop the mod q Quillen algebraic K-theory of normed algebras.
(3) To study the algebraic and topological bivariant K-theories and their relationship with Kasparov KK-theory for Z_2-graded complex and real algebras.

**Methodology :**
Methods of simplicial algebra, category theory, K-theory, algebraic topology and Banach algebras.

**Task Input:**
(1) Karoubi's conjecture on the isomorphism of algebraic and topological K-functors was confirmed by Higson for Karoubi-Villamayor algebraic K-functors and by Suslin and Wodzicki for Quillen algebraic K-functors. It was proved by H.Inassaridze for polynomial extensions of stable C*-algebras and by Kandelaki and H.Inassaridze for stable generalized operator algebras. (2) An algebraic K-theory of normed algebras was constructed by H.Inassaridze . Its relationship with topological K-theory and Karoubi-Villamayor K-theory of Banach algebras was established. During last twenty years many important works appeared in mod q theories. Neisendorfer studied a mod q homotopy theory and Browder investigated a mod q algebraic K-theory. Karoubi and Lambre introduced the mod q Hochschild homology and constructed the Dennis trace map from mod q algebraic K-theory to mod q Hochschild homology. Weibel constructed topologically a mod q Karoubi-Villamayor algebraic K-theory. Conduche, H. and N. Inassaridze investigated a mod q Tate-Farell-Vogel (co)homology of groups. (3) In noncommutative geometry one of the major interest is to finding topological invariants of some classes of noncommutative algebras. One of the powerful tool is Kasparov bivariant K-theory. The study of Kasparov KK-theory by methods of other mathematical theories may be considered as interesting problem. Kandelaki proved that Kasparov KK-theory may be expressed by algebraic, topological and Karoubi-Villamayor K-theories.

**Result, milestones :**
(1) Karoubi's conjecture on the isomorphism of algebraic and topological K-functors was confirmed by Higson for Karoubi-Villamayor algebraic K-functors and by Suslin and Wodzicki for Quillen algebraic K-functors. It was proved by H.Inassaridze for polynomial extensions of stable C*-algebras and by Kandelaki and H.Inassaridze for stable generalized operator algebras. (2) An algebraic K-theory of normed algebras was constructed by H.Inassaridze . Its relationship with topological K-theory and Karoubi-Villamayor K-theory of Banach algebras was established. During last twenty years many important works appeared in mod q

theories. Neisendorfer studied a mod q homotopy theory and Browder investigated a mod q algebraic K-theory. Karoubi and Lambre introduced the mod q Hochschild homology and constructed the Dennis trace map from mod q algebraic K-theory to mod q Hochschild homology. Weibel constructed topologically a mod q Karoubi-Villamayor algebraic K-theory. Conduche, H. and N. Inassaridze investigated a mod q Tate-Farell-Vogel (co)homology of groups. (3) In noncommutative geometry one of the major interest is to finding topological invariants of some classes of noncommutative algebras. One of the powerful tool is Kasparov bivariant K-theory. The study of Kasparov KK-theory by methods of other mathematical theories may be considered as interesting problem. Kandelaki proved that Kasparov KK-theory may be expressed by algebraic, topological and Karoubi-Villamayor K-theories.

### 4.2.1.2 Task Title : Equivariant (co)homology theories

Task coordinator : Hvedri Inassaridze, belonging to team: Tbilisi Team

**Objectives :**
To develop an equivariant (co)homology of groups of H.Inassaridze with application to algebraic K-theory.

**Methodology :**
Methods of homotopical algebra, particularly simplicial technique.

**Task Input:**

The task is depending on : Non-abelian (co)homology and derived functors
The study of groups with operators has many important applications in algebra and topology. Recent results obtained in equivariant stable homotopy theory, presented during a workshop at Stanford university and in equivariant algebraic K-theory. The origin of the equivariant investigation in homological algebra goes back to the work of J.H.C.Whitehead. Recently a cohomology theory of groups with operators was developed by Cegarra, motivated by the graded categorical groups classification problem suggested by Frohlich and Wall. The corresponding homology theory was treated by Cegarra and H.Inassaridze and an equivariant version of the classical Brauer-Hasse-Noether result was proved by Cegarra and Garzon.

**Result, milestones :**
Continuation of the investigation of the equivariant (co)homology of groups developed recently by H.Inassaridze. In particular, establishing the relationship of the equivariant cohomology of groups with higher equivariant extensions of groups, providing equivariant Hopf type formulas for higher equivariant integral group homology, proving the isomorphism of the third Quillen $K$-group of any ring with the third equivariant integer homology group of the Steinberg group of this ring under the action of the Steinberg group of the ring of integers. Developing that equivalent approach for Hochschild and cyclic homology theories.

### 4.2.2 Task Title : Non-abelian (co)homology and derived functors

Task coordinator : Nick Inassaridze, belonging to team: Tbilisi Team

**Objectives :**
To investigate n-fold Cech derived functors, generalised Hopf type formulas, non-abelian (co)homology and n-fold Cech (co)homology of open covers.

**Methodology :**

Methods and machinery of simplicial algebra, homology and homotopy theories and spectral sequences.

**Task Input:**

The task is depending on : Category theory, functor cohomology, Hochschild homology
The recent work of Donadze, N.Inassaridze and Porter introduced the notion of n-fold Cech derived functors of endofunctors on the category of groups, which resulted in obtaining the formulae of Hopf type for the non-abelian derived functors of the lower central series functors $Z_i(G)=G/\Gamma_i(G),i\geq 2$, where $\{\Gamma_i(G),i\geq 1\}$ is the lower central series of the group G. These correct and generalise the well known higher Hopf formulae of Brown-Ellis for integral group homology. Results of Pirashvili on Cech derived functors and on simplicial degrees of functors. Non-abelian (co)homology theories of Guin, H. and N. Inassaridze and others. Cech (co)homology and sheaf cohomology constructions.

**Result, milestones :**
See the subtasks descriptions.

#### 4.2.2.1 Task Title : N-fold Cech derived functors and Hopf type formulas

Task coordinator : Nick Inassaridze, belonging to team: Tbilisi Team

**Objectives :**
To study the properties of n-fold Cech derived functors of group valued functors. To develop the (co)homology theories of (pre)crossed modules. To derive the formulas of Hopf type for the higher CCG-homology of crossed modules and the higher homology of precrossed modules of Conduche-Ellis.

**Methodology :**
Methods and techniques of (pre)crossed modules and that of developed by Donadze, N.Inassaridze and Porter.

**Task Input:**

The task is depending on : Taylor tower for group valued functors
Work of Donadze, N.Inassaridze and Porter on n-fold Cech derived functors and generalised Hopf type formulas. Works on Cech derived functors having applications to classical group (co)homology theory, K-theory, non-abelian group homology and Conduche-Ellis homology of precrossed modules given by Pirashvili, H. and N. Inassaridze, Ladra, Khmaladze.
Initial works by Granjean, Carrasco and Cegarra on the (co)homology theory of crossed modules and by Conduche and Ellis on low dimensional homology of precrossed modules. Further investigations of these (co)homologies by Ladra, Pirashvili, N.Inassaridze and others.

**Result, milestones :**
To introduce the notion of n-fold Cech derived functors of group valued functors, generalising that of previously given by Donadze, N.Inassaridze and Porter. Investigation of the relation of the n-fold Cech derived functors with the non-abelian derived functors of group valued functors. In particular, obtain the results when functors are determined on a pointed variety of groups with operators and have simplicial degree 1. Giving some applications to the well known (co)homology functors , K-functors, non-abelian (co)homology of groups and Lie algebras.
To derive the generalised Hopf type formulas for the higher CCG-homology of

crossed modules and for the higher Conduche-Ellis homology of precrossed modules, implying some explicit calculations for these homologies in the third dimension using methods of Ellis and others for integral group homology.

#### 4.2.2.2 Task Title : Conjecture of Pirashvili

Task coordinator : Nick Inassaridze, belonging to team: Tbilisi Team

**Objectives :**
To solve or make substantial contributions towards the solution of the (weak) conjecture of Pirashvili on finiteness of simplicial degree of non-abelian derived functors.

**Methodology :**
Methods of simplicial algebra, derived functors and spectral sequences.

**Task Input:**
It will be dealed with algebraic models for n-types of simplicial groups and thus for connected (n+1)-types of spaces. The origin of such question goes back to the classical paper of Dold and Puppe. Pirashvili conjectured that if a functor from pointed variety of groups with operators to the category of groups has finite simplicial degree, then its non-abelian derived functors also have finite simplicial degree.

**Result, milestones :**
Solving or making a substantial progress about Pirashvili's (weak) conjecture.

#### 4.2.2.3 Task Title : Higher non-abelian cohomology of Lie algebras

Task coordinator : Hvedri Inassaridze, belonging to team: Tbilisi Team

**Objectives :**
To construct a higher non-abelian cohomology of Lie algebras and develop it in the whole context of non-abelian (co)homology theory of Lie algebras.

**Methodology :**
Methods of non-abelian homological algebra.

**Task Input:**

The task is depending on : Non-abelian (co)homology and derived functors
Initial works by Guin, in which the low dimensional non-abelian homology of groups and Lie algebras has been defined, based on the tensor product constructions of groups of Brown-Loday and of Lie algebras of Ellis. In the same works Guin introduced a low dimensional non-abelian cohomology of groups and Lie algebras. In further works of H. and N. Inassaridze the construction and investigation of a non-abelian (co)homology theory of groups in all dimensions have been given. In the recent work of Ladra, Khmaladze and N.Inassaridze the same non-abelian (co)homology theory for Lie algebras, but for cohomology up to the second dimension, has been developed.

**Result, milestones :**
Definition and development of a workable higher dimensional non-abelian cohomology of Lie algebras.

#### 4.2.2.4 Task Title : (co)homology of crossed structures

Task coordinator : D.Guin, belonging to team: Montpellier Team

**Objectives :**
To continue the study of the non abelian (co)homology in the general unifed framwork of crossed structures (crossed modules, crossed associatives algebras, crossed Lie algebras,...)

**Methodology :**
To use the methodology given by the general (co)homology theories associated to a comonad.

**Task Input:**

The task is depending on : Non-abelian (co)homology and derived functors
Several authors have given definitions of non abelian (co)homology for crossed modules or crossed Lie algebras. It seems that these definitions could be seen has particular cases of general situation.

**Result, milestones :**
To give a unique approach to study the non abelian (co)homology of different crossed structures.

#### 4.2.2.5 Task Title : N-fold Cech (co)homology of open covers

Task coordinator : Nick Inassaridze, belonging to team: Tbilisi Team

**Objectives :**
To construct and study an n-fold Cech (co)homology of open covers of spaces with coefficients in sheaves of abelian groups.

**Methodology :**
Methods of homology and homotopy theories and spectral sequences.

**Task Input:**

The task is depending on : N-fold Cech derived functors and Hopf type formulas
The constructions of Cech (co)homology and sheaf cohomology. Cech derived functors of group valued functors are algebraic analogues of the Cech (co)homology constructions of open covers of topological spaces with coefficints in sheaves of abelian groups.

**Result, milestones :**
Construction and study of n-fold Cech (co)homology of open covers of spaces with coefficients in sheaves, inspired by the inverse influence from algebra to topology. Investigation of this (co)homology theory, in connection with sheaf cohomology of spaces.

### 4.2.3 Task Title : Cohomology theories, cobordism and formal groups

Task coordinator : Adrew Baker, belonging to team: Glasgow Team

**Objectives :**
To study various aspects of generalized cohomology theory using modern stable homotopy theory, and more geometrical methods.

**Methodology :**
Methods of modern stable homotopy theory, cobordism, formal groups, periodic cohomology theories, Chern character, braid groups.

**Task Input:**
This is made precise in the subtasks descriptions but mainly the work of various mathematical groups in the past years.

**Result, milestones :**
See the subtask descriptions.

### 4.2.3.1 Task Title : Structured ring spectra and applications

Task coordinator : Andrew Baker, belonging to team: Glasgow Team

**Objectives :**
Recent work on strict multiplications on ring spectra making use of modern categories of spectra with strictly monoidal smash products have led to considerable reworking of "classical results" on ring spectra, leading to families of results on various Landweber exact and other theories (see the work of Strickland, Lazarev). Construction of tmf and other new spectra using rigidifications of diagrams of ring spectra have become of central importance in the study of elliptic spectra and related homotopy theory. Various approaches to obstruction theory for rigidification of maps are known and systematic development of and calculation with these will be undertaken based on the approach of Robinson and Whitehouse. We will focus particulary on periodic spectra such as those related to Johnson-Wilson spectra and elliptic cohomology.

Strictly commutative multiplications lead to extended power and other operations related to those of A. Baker (Cont. Math. 220 (1998). The application of these in cases such as elliptic cohomology should have interesting calculational uses. We will apply such operations in the elliptic cohomology of geometric objects such as toric manifolds. Panov and Ray have initiated the study of loops on such spaces, which will provide crucial input when combined with their expertise in complex oriented cohomology theory.

The use of formal group law theory pervades all of above.

**Methodology :**
Modern methods of stable homotopy theory, formal groups. $E_\infnty$ structure, $\Gamma$-homology.
This subtask involving lot of persons and which may create new collaboration is supposed to go on over all the duration of the network.

**Task Input:**

The task is depending on : Cohomology theories, cobordism and formal groups
See above.

**Result, milestones :**
New calculational results concerning in particular elliptic cohomology.

### 4.2.3.2 Task Title : Transfers, characteristic classes and cohomology rings

Task coordinator : Malkhaz Bakuradze, belonging to team: Tbilisi Team

**Objectives :**

(1) To produce new information on the multiplicative structure of the symplectic cobordism ring.
(2) To use transferred characteristic classes for more natural presentation of the complex oriented cohomology rings of finite groups.

**Methodology :**
Machinery of formal group laws, characteristic classes, tranfers and spectral sequences.

**Task Input:**
(1) Recently Bakuradze, Jibladze and Vershinin have produced new kind of relations in symplectic cobordism. They express divisibility certain characteristic classes by transferred classes. These relations have been shown to imply that for $n<5$ products of 4n dimensional generators by the Ray elements fall into the ideal generated by the Ray elements of much more lower dimension.
(2) Various finite groups are good in the sense that their complex oriented cohomology rings are generated by transferred Chern classes. Recently Bakuradze and Priddy completely determined the multiplicative structure for $\Sigma_p \wr U(1)$, the universal example, in various complex oriented comology theories. The principal task being to compute Chern classes of transferred bundle in terms of its transferred Chern roots.

**Result, milestones :**
(1) To place the relations of the above type in the context of Vershinin's and Kochman's tables and thus give multiplicative information in dimensions covered by these tables.
(2) Presentations of Morava K-theory and Brown-Peterson cohomology of p-groups in terms of transferred Chern classes.

### 4.2.3.3 Task Title : Study of generalisations of braids from algebraic and homological points of view.

Task coordinator : V.Vershinine, belonging to team: Montpellier Team

**Objectives :**
From the begining of 90-es motivated mainly by Low-Dimensional Topology there appeared natural generalizations of braid groups. In our work we are planning to continue the study of these generalizations from various points of view. For the classical braid groups there exist extremely rich algebraic theory, interesting homological properties and various applications: in Physics, Cryptography etc. We are planing to transfer some of these properties for generalizations as well as establish new ones.

**Methodology :**
Combinatorial group theory and algebraic topology.

**Task Input:**
Presentation of Birman, Ko, Lee of braids group, results of Jean Michel.

**Result, milestones :**
To obtain the analog of the Birman Ko Lee presentation for singular braid monoid.

### 4.2.4 Task Title : Cohomology and homotopy properties of manifolds

Task coordinator : Y. Félix, belonging to team: LLN Team

**Objectives :**
To study various properties of the homotopy and cohomology of manifolds as well as from their free loop spaces.

A sub-field of research is To investigate the l_p-cohomology of discrete groups and noncompact hyperbolic manifolds.

Also to work on the remaining case of the Vigue-Sullivan conjecture about the existence of infinitely many geodesics.

**Methodology :**
See the subtasks.

**Task Input:**
The work of Gromov, Goldshtein, Kuz'minov, and Shvedov. Preceding work of Vigue, Saneblidze, Hess, ... The Goodwillie calculus of functors.

**Result, milestones :**
See the subtasks.

#### 4.2.4.1 Task Title : L_p-cohomology of discrete groups and Riemannian manifolds

Task coordinator : Yaroslav Kopylov, belonging to team: Novosibirsk Team

**Objectives :**
To investigate the l_p-cohomology of discrete groups and noncompact hyperbolic manifolds.

**Methodology :**
Methods of the L_p-theory of differential forms on Riemannian manifolds. Methods of the theory of discrete transformation groups in Euclidean and hyperbolic spaces. l_p-Methods in the geometry of discrete groups and simplicial complexes.

**Task Input:**
The Davis-Okun vanishing theorems for the l_2-(co)homology of right-angled Coxeter groups. The Davis-Leary theorems on the l_2-cohomology of Artin groups. Gromov's results on l_p-cohomology of simplicial complexes. Theorems by Goldshtein, Kuz'minov, and Shvedov on the L_p-cohomology of warped products of Riemannian manifolds.

**Result, milestones :**
Obtaining analogues to the Davis-Okun theorems for right-angled Coxeter groups and the Davis-Leary formula for the l_2-cohomology of an Artin group. Finding conditions of triviality for the second (reduced and nonreduced) l_p-cohomology space of a discrete group. Applications to noncompact manifolds.

#### 4.2.4.2 Task Title : Embedding of manifolds

Task coordinator : Pascal Lambrechts, belonging to team: LLN Team

**Objectives :**
Determine the homotopy type of embedding spaces of manifolds into others.

**Methodology :**
Use a combination of Goodwillie calculus and rational homotopy type methods.

**Task Input:**

The task is depending on : Structure of functor categories, cohomology of functors
The team in Louvain-La-Neuve is one of the most performant in the world concerning the use of rational homotopy theory for geometric applications.
The team has also taken advantage of the visit of some experts in Goodwillie calculus to improve its ability to use the machinery.

**Result, milestones :**
Preliminary results have already been obtained by P. Lambrechts in collaboration with D. Stanley and L. Vandembroucq.

### 4.2.4.3 Task Title : Loop space homology of manifolds

Task coordinator : Y. Félix, belonging to team: LLN Team

**Objectives :**
Obtain results on the homology of loop spaces and free loop spaces by using the Chas and Sullivan construction and the Getzler-Jones connection with Hochschild homology.

**Methodology :**
Use methods for differential homological algebra and rational homotopy theory in order to be able to obtain properties of the Chas-Sullivan new operations.

This subtask involving lot of persons and which may create new collaboration is supposed
to go on over all the duration of the network.

**Task Input:**
Some preliminay steps have already been realized in collaboration with L. Menichi, J.C. Thomas and M. Vigué (Paris).

**Result, milestones :**
Y. Felix, J.C. Thomas and M. Vigué have given a model for the free loop space depending only on the Adams-Hilton model of the space. This model enables the computation of the Gerstenhaber algebra on the homology of the free loop space.

The next step is to express in the model the connecting map that makes the loop homology a Batalin-Vilkovisky algebra. This construction will lead to the computation of the equivariant free loop space of the manifold.

The Gerstenhaber isomorphism between the homology of the free loop space and the Hochschild homology of the cochains suggests that the relation between equivariant free loop space and Cyclic homology must preserve some extra structure related to the Chas-Sullivan operations.

It has been conjectured that for hyperbolic manifolds, the homology of the free loop space has an exponential growth. This is clearly related to the growth of the number of geometrically distinct closed geodesics on the manifold. We hope that the present methods will bring new tools to study the conjecture.

### 4.2.5 Task Title : Cohomology of algebraic groups

Task coordinator : N.Vavilov - I.Panin, belonging to team: St.-Petersburg Team

**Objectives :**
To study cohomology of (groups of points) of reductive algebraic groups and their subgroups over various classes of rings with the specific goal of generalising Suslin's rigidity and vanishing theorems to other groups, other coefficients and other cohomology theories. To make a significant progress towards a solution of the Milnor-Friedlander conjecture comparing abstract and continuous cohomology of reductive groups. To develop K-theory of exceptional algebraic groups in its interplay with structure theory and representation theory.

**Methodology :**
We propose to use methods of the theory of algebraic groups, oriented cohomology theories, representation theory, algebraic K-theory, algebraic homotopy theory, and formal groups. Some more specialised recent methods are mentioned in subtask descriptions.

**Task Input:**
The work of various research groups in the U.S., France, U.K., Russia, Germany, Japan, Norway, Switzerland and some other research centres. See the subtask description for detailed survey of the known results.

**Result, milestones :**
Within 3 years we plan to publish at least 15-20 papers and preprints addressing these problems. See the subtask description for specific lists of expected results.

### 4.2.5.1 Task Title : Cohomology of groups of Lie type

Task coordinator : N. Vavilov, belonging to team: St.-Petersburg Team

**Objectives :**
(1) Generalise Suslin's vanishing theorem to higher cohomology of Chevalley groups.
(2) Study cohomology of generalised congruence subgroups in Chevalley groups over Henselian local rings using Suslin spectral sequence for various pairs of subgroups.
(3) Calculate cohomology of Steinberg groups modeled on Chevalley groups over a commutative ring with coefficients in fundamental and other small modules.

**Methodology :**
Apart from the general methods listed in the task methodology we plan to use some specific results such as Suslin spectral sequence for bi-crossed products of groups. The interrelation of the proposed objectives is as follows. Our goal is the proof of (1) and its generalisations which would form a decisive step towards a solution of Milnor-Friedlander problem discussed in subtask 5.2. With this end in mind we propose to use Suslin spectral sequence for generalised congruence subgroup presented as a product of a parabolic subgroup and the relative unipotent radical of the opposite parabolic (objective (2)). For the calculation of this spectral sequence it is instrumental to calcualte the cohomology of Chevalley groups with
coefficients in the internal Chevalley modules (which usually are fundamental modules, or exceptionally other modules with small highest weights). This explains our interest in Objective (3).

**Task Input:**
Suslin studied homology stability for the case of $GL_n$. He constructed a new spectral sequence generalising Hochschield-Serre spectral sequence, which allowed him to prove rigidity theorems, vanishing of cohomology and Milnor-Friedlander conjecture in some important cases.
There were literally hundreds of works (starting with the papers of Higman, Cline, Parshall Scott, van der Kallen, Bell, and many others) dedicated to the calculation of cohomology of Chevalley groups over fields with coefficients in some small modules (natural, microweight, fundamental, etc.). Here the answer is known over finite (sometimes over arbitrary) fields, but usually not over rings. In recent papers Vavilov and Petrov have noticed that the usual calculation of cohomology of these modules depend only on Steinberg relations and for large enough rank can be carried over to arbitrary commutative rings. On the other hand there are dozens of papers dedicated to the calculation of (co)homology of principal congruence subgroups with constant coefficients. Since the unipotent radicals of the parabolic subgroups arising in our calculations are usually non-abelian, one has to look at the factors of their descending central series, so there should be a non-trivial interrelation with Task 2.

**Result, milestones :**
(1) Generalise Suslin's vanishing theorem for $H^3$ from the case of $SL_2$ to all Chevalley groups.
(2) Calculate (non-abelian) cohomology of Levi factors of parabolic subgroups on unipotent radicals, and on internal Chevalley modules.
(3) Explicitly calculate Suslin spectral sequence for various types of bi-crossed products of congruence-like subgroups in Chevalley groups over local rings with the view of generalising Suslin vanishing theorem to higher cohomology.

4.2.5.2 **Task Title : Rigidity and Milnor-Friedlander conjecture**
Task coordinator : I. Panin, belonging to team: St.-Petersburg Team

**Objectives :**
To extend Suslin's rigidity theorem to all cohomology theories represented by a T-spectrum in the sense of Voevodsky. To make a significant contribution towards the proof of the Milnor-Friedlander conjecture

**Methodology :**
$A^1$-homotopy theory of Morel-Voevodsky, oriented cohomology theories, formal group laws, Thom-Gysin homomorphisms

**Task Input:**
Suslin's rigidity theorem states that an inclusion $k < K$ of algebraically closed fields induces an isomorphism on Quillen's K-groups with finite coeffitients. Later Gillet, Thomason and Gabber established similar results in the case of strict Henselization at a smooth point of a smooth algebraic variety and for Henselian pairs. The general linear case of the Milnor-Friedlander conjecture follows from these results. In 1996 Suslin and Voevodsky extended the rigidity results replacing K-functor by a homotopy invariant functor with transfers. Yagunov and Panin extended Suslin's rigidity theorem to the case of algebraic cobordism and more generally to any orientable cohomology theory on smooth algebraic varieties.

**Result, milestones :**
(1) to prove the rigidity in the Henselian case for all cohomology theories represented by a T-spectrum;
(2) to prove strict homotopy invariance of the Nisnevich Witt sheaf;

(3) to construct trace homomorphisms and the fundamental class of a smooth projective variety for an oriented homology theory;
(4) to use these results in order to contribute towards the proof of the Milnor-Friedlander conjecture

**4.2.5.3 Task Title : Nonabelian K-theory and structure of classical and exceptional groups.**

Task coordinator : A. Stepanov, belonging to team: St.-Petersburg Team

**Objectives :**
(1) Generalize Vaserstein-van der Kallen-Kolster prestabilisation results, as well as results of Suslin and Bak on the behaviour in the metastable range, to other Chevalley groups.
(2) Extend the van der Kallen - Tulenbaev centrality theorem for K_2 to the exceptional Chevalley groups of types E_6, E_7, E_8, and F_4, over an arbitrary commutative ring and possibly to some twisted Chevalley groups.
(3) Develop Volodin's K-theory for exceptional groups, prove analogues of Suslin's comparison theorem for Volodin's and Quillen's K-theories for other types of groups, generalise Suslin's stability theorem to Volodin's K-theory modeled on reductive groups.
(4) Construct an algebraic version of mod q K_2, generalising Milnor's classical definition, previously defined by Browder using topological means.

**Methodology :**
We plan to use the usual general methods of the structure theory, arithmetic theory and representation theory of algebraic groups, algebraic K-theory and simplicial algebra. We envisage that most of our specific calculations will be based on the geometry of minimal modules and such recently developed methods on non-abelian K-theory and structure theory as localisation-completion, decomposition of unipotents, and multiple commutator calculus.

**Task Input:**
The starting point here are the works of Bass, Milnor, Serre, Bak, Suslin, Dennis, van der Kallen, Wilson, Golubchik, Vaserstein, Browder, Kolster and others on algebraic K-theory and its application to structure theory, specifically on the stability, prestability and qualitative behaviour of classical groups in the meta-stable range. These results were generalized to Chevalley groups by Matsumoto, Stein, Abe, Suzuki and others. The works by Matsumoto and Stein introduced representation theoretic methods for calculations in exceptional Chevalley groups, further developed in joint works of Vavilov, Plotkin, Stepanov and others. Semenov, Plotkin and Vavilov tabulated weight diagrams of basic representations of Chevalley groups and started a computer study of these modules, Vavilov studied geometry of minimal modules for these groups. On the other hand very little is known about homology stability/stability of higher K-functors for exceptional groups. For classical groups these questions were essentially solved by van der Kallen, Suslin, Panin, Habdank, and others.

**Result, milestones :**
(1) Define analogues of Vaserstein tilde-elementary groups in terms of ESD and Freudenthal transvections, and compute the kernel of maps K1(\Delta,R)→K1(\Phi,R) in the metastable range
(2) Generalise Suslin-Tulenbaev's theorem on K_2 stability to all regular embeddings of Chevalley groups. Prove analogues of Kolster prestability theorem for all Chevalley groups.
(3) Establish stability for non-simply connected groups, define analogues of L-groups modeled on exceptional groups and prove analogues of Karoubi theorem

for exceptional embeddings.
(4) Extract important corollaries for the structure theory of (groups of points) of reductive groups over arbitrary commutative rings, such as, e.g. description of subgroups normalised by a part of elementary matrices (tensored subgroups, subring subgroups, ring extension subgroups and other Aschbacher classes).
(5) Milnor type definition of mod q K2 will be used to established the relationship with various mod q symbol groups generalising Matsumoto, Dennis, Stein, van der Kallen and Kolster's results.

### 4.2.6 Task Title : Category theory, functor cohomology, Hochschild homology

Task coordinator : L. Schwartz, belonging to team: Paris 13 Team

**Objectives :**
The object of this task is to study various aspects of functor categories as well as related cohomology theories. The category themselves or particular objects, or cohomology properties will be the object of study. Another goal will be to extend known result in the case of abelian category to more general cases.

**Methodology :**
Mainly homological algebra, simplicial topology, representation theory.

**Task Input:**
This is made precise in the subtasks descriptions but mainly the work of various mathematical groups in the past years.

**Result, milestones :**
See the subtasks descriptions.

### 4.2.6.1 Task Title : Taylor tower for group valued functors

Task coordinator : N. Inassaridze, belonging to team: Tbilisi Team

**Objectives :**
To construct and investigate the Taylor tower for group valued functors.

**Methodology :**
Tools of homotopical algebra and topology.

**Task Input:**

The task is depending on : Non-abelian (co)homology and derived functors
The notion of crossed effect of functors in the non-abelian framework introduced by Baues and Pirashvili generalising the classical notion of Eilenberg and MacLane. Papers of Johnson and McCarthy on Taylor towers for functors of additive categories.

**Result, milestones :**
Definition of Taylor tower for functors from the pointed category to the category of groups. Study the limit of the tower and determination of sufficient conditions for its convergence. Construction the Taylor tower for some known functors such as general linear group functor, Steinberg group functor, K-functors, cyclic (co)homology functors, non-abelian tensor products and (co)homology of groups and Lie algebras.

### 4.2.6.2 Task Title : Hochschild and cyclic (co)homology, Hopf algebras

Task coordinator : M. Vigue, belonging to team: Paris 13 Team

**Objectives :**

(1) To continue the investigation of primary Hochschild and cyclic (co)homology for commutative algebras.
(2) To express cyclic (co)homology as cotriple (co)homology, perhaps in particular cases.
(3) To continue the study of the Hochschild cohomology of triangular algebras and the relations with the representation theory of algebras. To extend this studie to the Hochschild-Mitchell cohomology
of a category with finitely many objects
(4) (J. M. Oudom) 1-) In 2001, I. Moerdijck proposes a family of Hopf algebra structures on the rooted trees
which does include the Connes-Kreimer Hopf algebra. The linear dual of the Connes-Kreimer Hopf algebra
is isomorphic to the envelopping algebra of the rooted trees Lie algebra. What does happen with the
duals of the Moerdijck Hopf algebras ?

As this subtask contains itself various objectives the duration will
be over all the period.

**Methodology :**

Methods of classical homological algebra, non-abelian derived functors, cyclic homology, DG-algebras and K-theory.

To use the methods developed by Bendiffalah-Guin for to study the Hochschild cohomology of triangular algebras and extend it to cohomology of categories.G

**Task Input:**

The task is depending on : Loop space homology of manifolds
(1) The classical (co)homology theories, in particular Hochshcild, de Rham and cyclic homology theories, primary K-theory of Browder, joint work of Loday and Quillen, works of Villamayor, Cortinas, J.A.Guccione, J.J.Guccione, Larsen, Lindenstrauss and others. (2) The classical joint papers by Barr and Beck on cotriple (co)homology theories and book of Loday on cyclic homology.

**Result, milestones :**

(1) Recently H. and N. Inassaridze constructed primary cyclic (co)homology and established some basic properties for primary Hochschild and cyclic (co)homology. Further development of theses theories and explicit computations of primary cyclic (Hochschild) homology for commutative algebras. Expression of the cyclic and primary cyclic (co)homology as cotriple (co)homology.

(2) Applications to representation theory of algebras and funtoriality of hochschild cohomology of algebras.

### 4.2.6.3 Task Title : Exact couples in Raikov-semiabelian categories

Task coordinator : Yaroslav Kopylov, belonging to team: Novosibirsk Team

**Objectives :**

To develop the foundations of the theory of exact couples in Raikov-semiabelian categories.

**Methodology :**
Methods of homological albgebra in almost abelian categories.

**Task Input:**
The Eckmann-Hilton approach to derivation of exact couples in abelian categories.Results on exactness of the Ker-Coker-sequence
and cohomology sequence in a semiabelian category by Kopylov&Kuz'minov and Glotko&kuz'minov. THeorems by Kuz'minov&Cherevikin on Raikov's axioms of a semiabelian category.

**Result, milestones :**
Construction of the spectral sequence corresponding to an exact couple with strict endomorphism in a semiabelian category. Description of the properties of the spectral sequence for a Rees system.

**4.2.6.4 Task Title : Structure of functor categories, cohomology of functors**
Task coordinator : L. Schwartz, belonging to team: Paris 13 Team

**Objectives :**
The properties of study are quasi-coherence of
certain
functor (abelian) categories, as well as the Krull-Gabriel filtration. More generally certain of these categories are expected to be artinian. A consequence would finitness properties of injective resolutions under nice assumptions.
injective resolutions are expected

**Methodology :**
Methods of homological algebra, representation theory, unstable modules over the Steenrod algebra and combinatorics.

**Task Input:**
Partial results of G. Powell, L. Schwartz, V. Franjou, N. Kuhn.

**Result, milestones :**
The main expected result is a better comprehension
of global properties of the above mentionned categories and ofthe closely related category of unstable module. Implicit in this
problem is the question of how much of the unstable module structure is determined by the Poincaré
séries (under appropriate hypothesis on the module!).

**4.2.6.5 Task Title : Homologies of categories and rewriting systems**
Task coordinator : D.Guin, belonging to team: Montpellier Team

**Objectives :**
The Squier-Kobayashi's criterion give a homological (finitness) condition to characterise some geometrical properties of calculus in rewriting system of words. The objective is to extend this criterion in more complexes paradigm of calculus, like rewriting of terms

**Methodology :**
To describe an analogue of the Squier's resolution using "critical pairs" in the general framework of the Hochschild-Mitchell's and Baues's cohomologies of categoriess

**Task Input:**
See above.

**Result, milestones :**
This work is a crucial step to understand the problem of the comparaison of programs.

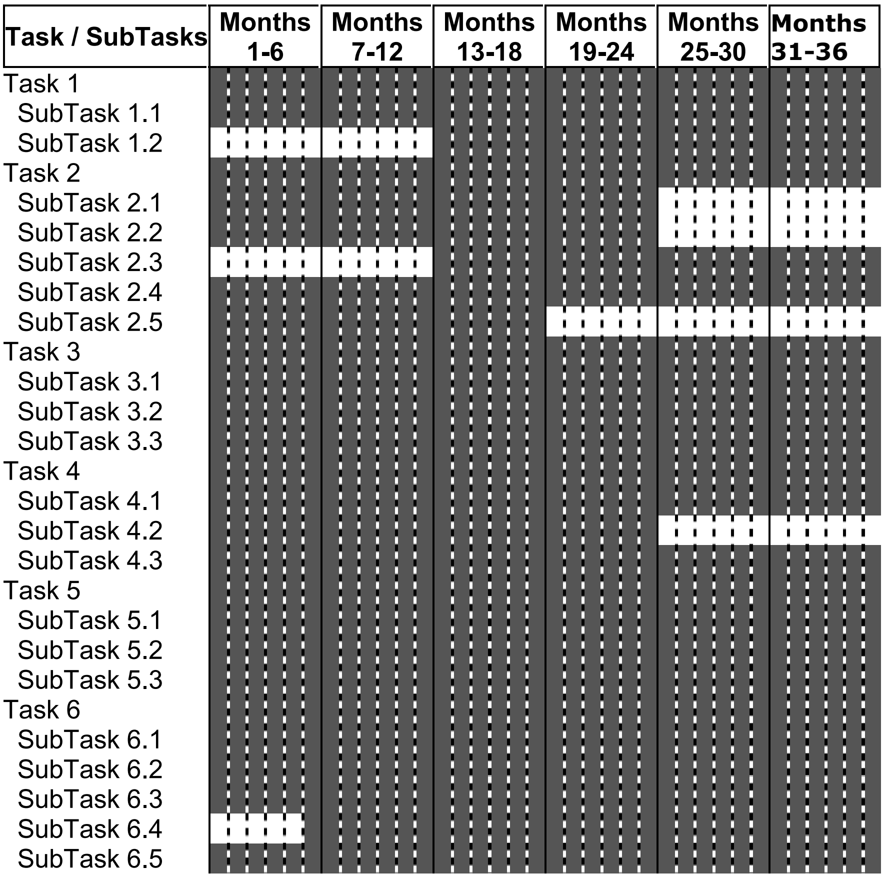
## 4.3 Project Management

### 4.3.1 Planning & Task allocation

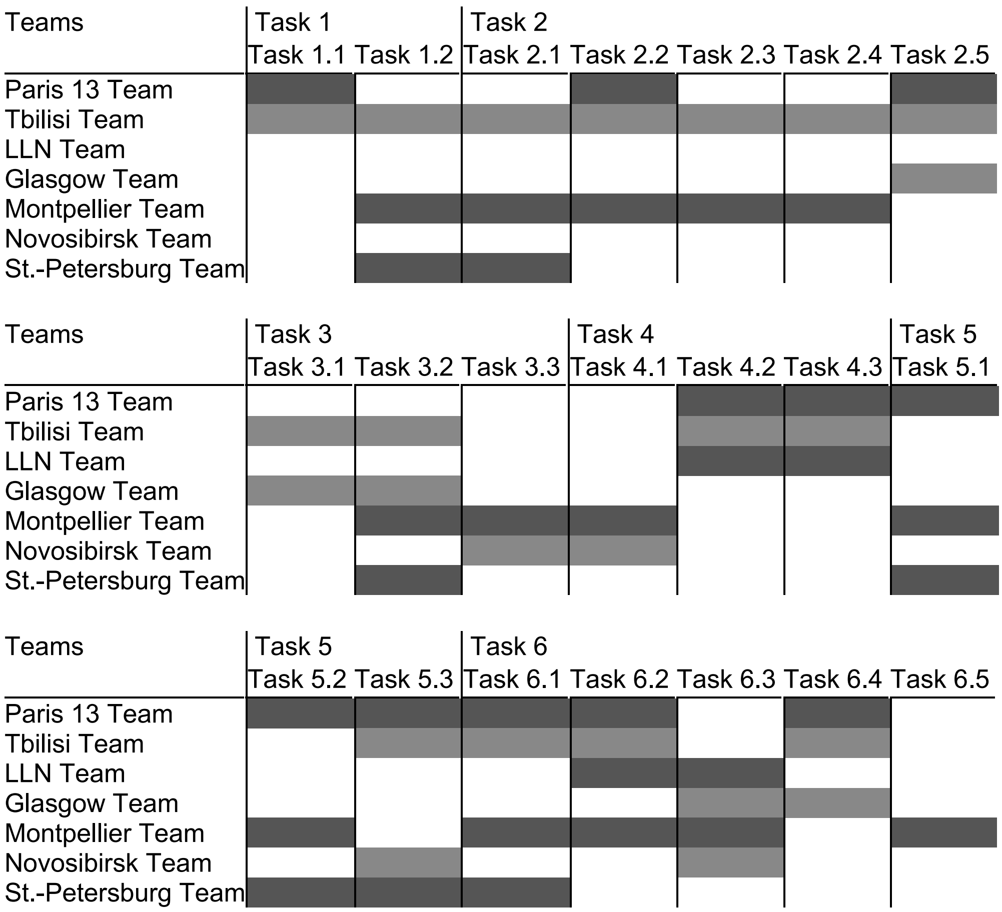
#### 4.3.1.1 List of Task Titles

1. Algebraic K-theory
   - 1.1 K-theory of normed algebras
   - 1.2 Equivariant (co)homology theories
2. Non-abelian (co)homology and derived functors
   - 2.1 N-fold Cech derived functors and Hopf type formulas
   - 2.2 Conjecture of Pirashvili
   - 2.3 Higher non-abelian cohomology of Lie algebras
   - 2.4 (co)homology of crossed structures
   - 2.5 N-fold Cech (co)homology of open covers
3. Cohomology theories, cobordism and formal groups
   - 3.1 Structured ring spectra and applications
   - 3.2 Transfers, characteristic classes and cohomology rings
   - 3.3 Study of generalisations of braids from algebraic and homological points of view.
4. Cohomology and homotopy properties of manifolds
   - 4.1 L_p-cohomology of discrete groups and Riemannian manifolds
   - 4.2 Embedding of manifolds
   - 4.3 Loop space homology of manifolds
5. Cohomology of algebraic groups
   - 5.1 Cohomology of groups of Lie type
   - 5.2 Rigidity and Milnor-Friedlander conjecture
   - 5.3 Nonabelian K-theory and structure of classical and exceptional groups.
6. Category theory, functor cohomology, Hochschild homology
   - 6.1 Taylor tower for group valued functors
   - 6.2 Hochschild and cyclic (co)homology, Hopf algebras
   - 6.3 Exact couples in Raikov-semiabelian categories
   - 6.4 Structure of functor categories, cohomology of functors
   - 6.5 Homologies of categories and rewriting systems

**4.3.1.2 The project will last 36 months with the activities as indicated in the diagram below**



#### 4.3.1.3 Team involvement




### 4.3.2 Project Management Description

In order to organize exchanges and discussions in the network various activities are planned. They are of four different types :

1). Short terms visit of a meember of a team
in an another team. These visits are supposed to be supported by the network and there should be each year around 10 such visits. The aim of such visits will mainly be to the developement of a cooperation between two (or more) members
members of the network. The priority will be given for advanced projects and well established cooperation. A certain number (but smaller)
of visits
can be supported to support new cooperation.
This will be decide by the team leaders.

2). Long term visits : this will not be supported directly by the network, but for particularly important cooperation teams will be encouraged to look for other source of fundings. No number of such visits is planned but one can expect a few ones over the duration of the network.

3). Congresses, so that most members of the network could meet. Two are

expected over the 3 years, possible locations are
Tbilisi, Montpellier, Paris, Saint-Petersbourg, Glasgow. Even if this congress will try to attract a majority of members of the members it is proposed to choose a main theme for each congress such that :
- algebraic K-theory
- functor categories and cohomology, non-abelian cohomology,
- cobordism theory and elliptic cohomology

The duration would be 4 days.

3). One "summer school", with intensive course on the main subjects of the network, for students. Duration should one week or two weeks. Possible location is the Centre International de Recherche Mathématique (CIRM) in Marseille-Luminy.

To orgnize all that 3 team leaders meetings will organized (they may coincide with short term visit).

Most talks are supposed go on all over the 36 months. Indeed except for special reasons (such as long term planned visits) and by the nature of mathematical research, progress on a subject may occur at any time.

Possible schedule is :
end of 2004 : first meeting
spring 2005 : "summer scholl"
summer 2006 : second meeting

H. Inassaridze will be in charge of coordinating the NIS team.

### 4.4 Project costs

### 4.4.1 Cost Table

The breakdown of costs of the INTAS contribution (in EURO) is given in the tables below.

| INTAS MEMBER STATE TEAMS | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Team name | Cost categories | | | | | | TOTAL |
| | | Labour Costs | Overheads | Travel & subs. | Consumables | Equipment | Other | (EURO) |
| 1 | Paris 13 Team... | 0 | 2000 | 10000 | 0 | 0 | 0 | 12000 |
| 2 | LLN Team | 0 | 1600 | 8000 | 0 | 0 | 0 | 9600 |
| 3 | Glasgow Team | 0 | 2000 | 10000 | 0 | 0 | 0 | 12000 |
| 4 | Montpellier T... | 0 | 1600 | 8000 | 0 | 0 | 0 | 9600 |
| SUBTOTAL | (EURO) | 0 | 7200 | 36000 | 0 | 0 | 0 | 43200 |

| NIS TEAMS | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Team name | Cost categories | | | | | | TOTAL |
| | | Labour Costs | Overheads | Travel & subs. | Consumables | Equipment | Other | (EURO) |
| 5 | Tbilisi Team | 46800 | 0 | 6200 | 0 | 2000 | 0 | 55000 |
| 6 | Novosibirsk T... | 21600 | 0 | 2400 | 0 | 1000 | 0 | 25000 |
| 7 | St.-Petersbur... | 47000 | 0 | 8000 | 0 | 0 | 0 | 55000 |
| SUBTOTAL | (EURO) | 115400 | 0 | 16600 | 0 | 3000 | 0 | 135000 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| TOTAL | (EURO) | 115400 | 7200 | 52600 | 0 | 3000 | 0 | 178200 |

### 4.4.2 Justification of Costs

#### 4.4.2.1 Labour costs (only for NIS teams)

<u>Team name: Tbilisi Team</u>

| Number of individual grants | | Cost per month | Total number of man months | Total cost (EURO) |
|---|---|---|---|---|
| Team Leader | 1 | 350 | 36 | 12600 |
| Senior Researcher | 3 | 280 | 108 | 30240 |
| Scientist / Engineer | 1 | 110 | 36 | 3960 |
| Technical or Other | 0 | 0 | 0 | 0 |
| TOTAL | | | | 46800 |

<u>Team name: Novosibirsk Team</u>

| Number of individual grants | | Cost per month | Total number of man months | Total cost (EURO) |
|---|---|---|---|---|
| Team Leader | 1 | 350 | 36 | 12600 |
| Senior Researcher | 0 | 0 | 0 | 0 |
| Scientist / Engineer | 1 | 250 | 36 | 9000 |
| Technical or Other | 0 | 0 | 0 | 0 |
| TOTAL | | | | 21600 |

<u>Team name: St.-Petersburg Team</u>

| Number of individual grants | | Cost per month | Total number of man months | Total cost (EURO) |
|---|---|---|---|---|
| Team Leader | 1 | 300 | 30 | 9000 |
| Senior Researcher | 3 | 250 | 80 | 20000 |
| Scientist / Engineer | 4 | 150 | 120 | 18000 |
| Technical or Other | 0 | 0 | 0 | 0 |
| TOTAL | | | | 47000 |

#### 4.4.2.2 Travel and subsistence

<u>Team 1 (Paris 13 Team)</u>
This cost will support one or two misions per year of members of the node in Tbilissi or Novossibirsk.

<u>Team 2 (LLN Team)</u>
This cost will support one or two misions per year of members of the node in Tbilissi or Novosibirsk.

<u>Team 3 (Glasgow Team)</u>
This cost will support one or two misions per year of members of the node in Tbilissi or Novosibirsk.

<u>Team 4 (Montpellier Team)</u>
This cost will support one or two misions per year of members of the node in Tbilissi or Novosibirsk.

<u>Team 5 (Tbilisi Team)</u>

<u>Team 6 (Novosibirsk Team)</u>

Team 7 (St.-Petersburg Team)

#### 4.4.2.3 Consumables

Team 1 (Paris 13 Team)

Team 2 (LLN Team)

Team 3 (Glasgow Team)

Team 4 (Montpellier Team)

Team 5 (Tbilisi Team)

Team 6 (Novosibirsk Team)

Team 7 (St.-Petersburg Team)

#### 4.4.2.3 Equipment

Team 1 (Paris 13 Team)

Team 2 (LLN Team)

Team 3 (Glasgow Team)

Team 4 (Montpellier Team)

Team 5 (Tbilisi Team)

Team 6 (Novosibirsk Team)

Team 7 (St.-Petersburg Team)

#### 4.4.2.4 Other Costs

Team 1 (Paris 13 Team)

Team 2 (LLN Team)

Team 3 (Glasgow Team)

Team 4 (Montpellier Team)

Team 5 (Tbilisi Team)

Team 6 (Novosibirsk Team)

Team 7 (St.-Petersburg Team)

### 4.5 Project innovation potential and dissemination of results

Inside mathematics this program is located at a place where various particular fields meet: homotopical algebra, homological algebra, K-theory, generalized cohomology which are all parts of algebraic topology but also topology and geometry of manifolds, representation theory of abstract and algebraic groups, category theory and combinatoric. Interesting and important new results are expected in the described subfields, but also it is hoped that new results and concepts will emerge from the relations between all these subfields (which has soon been the case in the recent years).

If this kind of program has no immediate applications outside mathematics, it is an essential part to the general research effort in exact sciences and as such contributes to the general progress of knowledge.